Physical Optics

Deformable Mirrors

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Astro 289, UCSC
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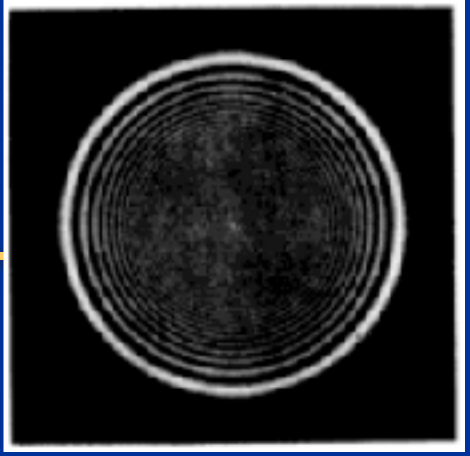
(slides from Claire Max)

Levels of Models in Optics



Geometric optics - rays, reflection, refraction

↓

Physical optics (Fourier optics) - diffraction, scalar waves

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Electromagnetics - vector waves, polarization

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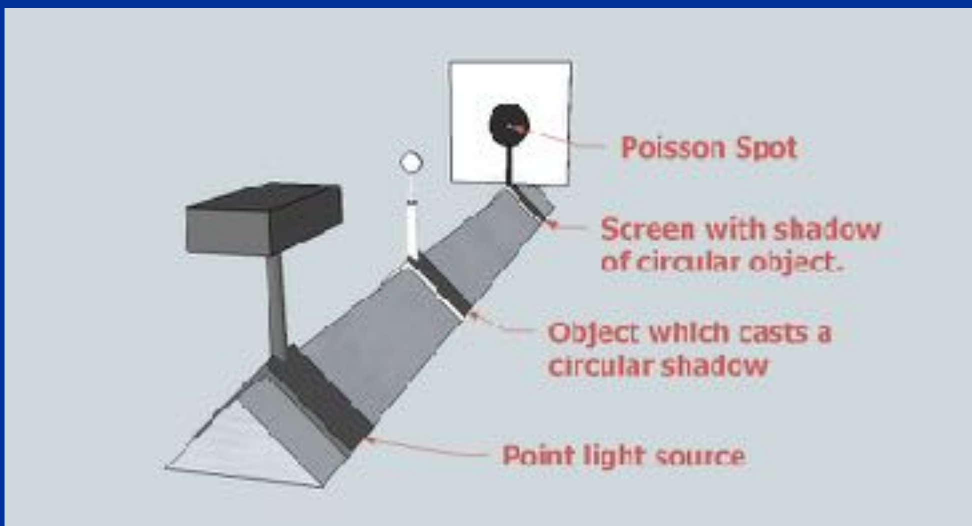
Quantum optics - photons, interaction with matter, lasers

Part 2: Fourier (or Physical) Optics

Arago's (or Fresnel, or Poisson) spot



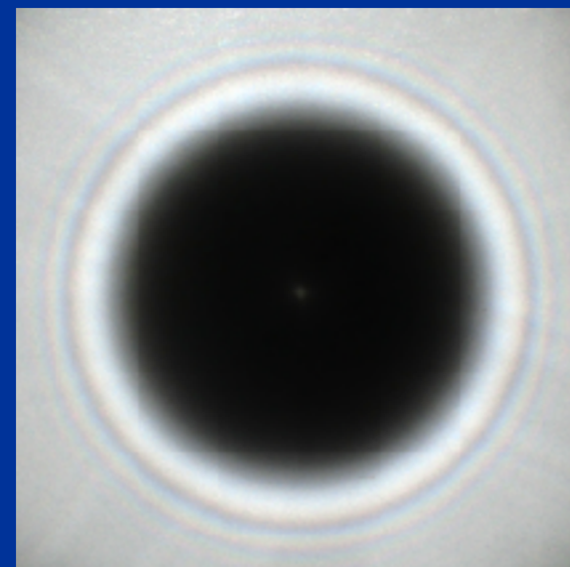


photo of the shadow of a 5.8 mm obstacle
(from wikipedia.org)

Maxwell's Equations: Light as an electromagnetic wave (Vectors!)

$$\nabla \cdot \vec{E} = 4\pi\rho$$

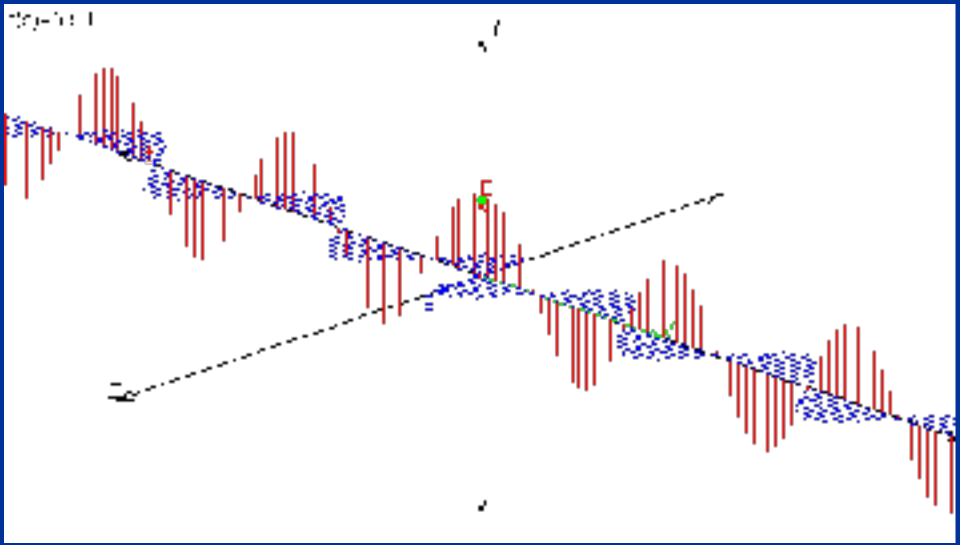
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

Light as an EM wave

- Light is an electromagnetic wave phenomenon, **E** and **B** are perpendicular
- We detect its presence because the EM field interacts with matter (pigments in our eye, electrons in a CCD, …)



Physical Optics is based upon the scalar Helmholtz Equation (no polarization)

- In free space

$$\nabla^2 E_\perp = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_\perp$$

- Traveling waves

$$E_\perp(x,t) = E_\perp(0, t \pm x/c)$$

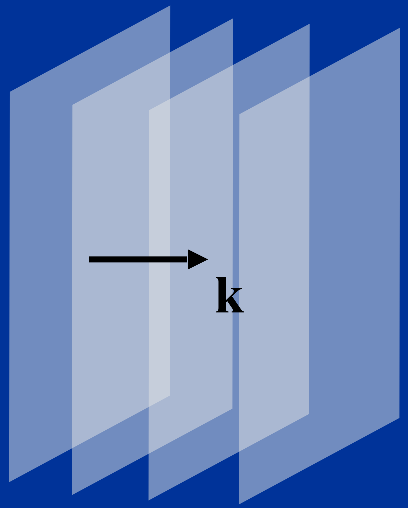
- Plane waves

$$E_\perp(\mathbf{x},t) = \tilde{E}(\mathbf{k}) e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})}$$

Helmholtz Eqn.,
Fourier domain

$$k^2 \tilde{E} = (\omega / c)^2 \tilde{E}$$

$$k = \omega / c$$



Dispersion and phase velocity

- **In free space** $k = \omega / c$ where $k \equiv 2\pi/\lambda$ and $\omega \equiv 2\pi\nu$
  - Dispersion relation $k(\omega)$ is linear function of ω
  - Phase velocity or propagation speed = $\omega / k = c = const.$
- **In a medium**
  - Plane waves have a *phase velocity*, and hence a wavelength, that depends on frequency

$$k(\omega) = \omega / v_{phase}$$

  - The "slow down" factor relative to c is the *index of refraction*, $n(\omega)$

$$v_{phase} = c / n(\omega)$$

Optical path - Fermat's principle

- Huygens' wavelets
- Optical distance to radiator:

$$\Delta x = \mathrm{v}\,\Delta t = \mathrm{c}\,\Delta t / n$$

$$\mathrm{c}\,\Delta t = n\,\Delta x$$

$$\text{Optical Path Difference} = \text{OPD} = \int n\,\mathrm{d}x$$

- Wavefronts are iso-OPD surfaces
- Light ray paths are paths of least* time (least* OPD)

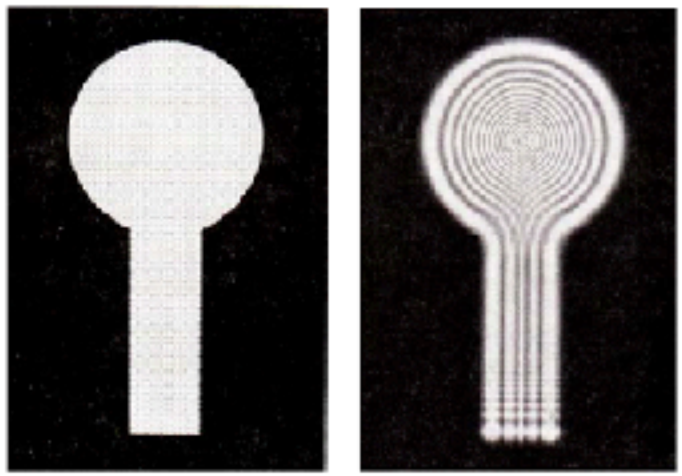
*in a local minimum sense

What is Diffraction?

When an opaque body is placed midway between an observing screen and a point source, diffraction effects produce an intricate shadow made up of bright and dark regions quite unlike anything one might expect from the principles of geometrical optics.

The phenomenon of *diffraction* has thus been defined as "any deviation of light rays from rectilinear paths that cannot be interpreted as reflection or refraction".

Aperture

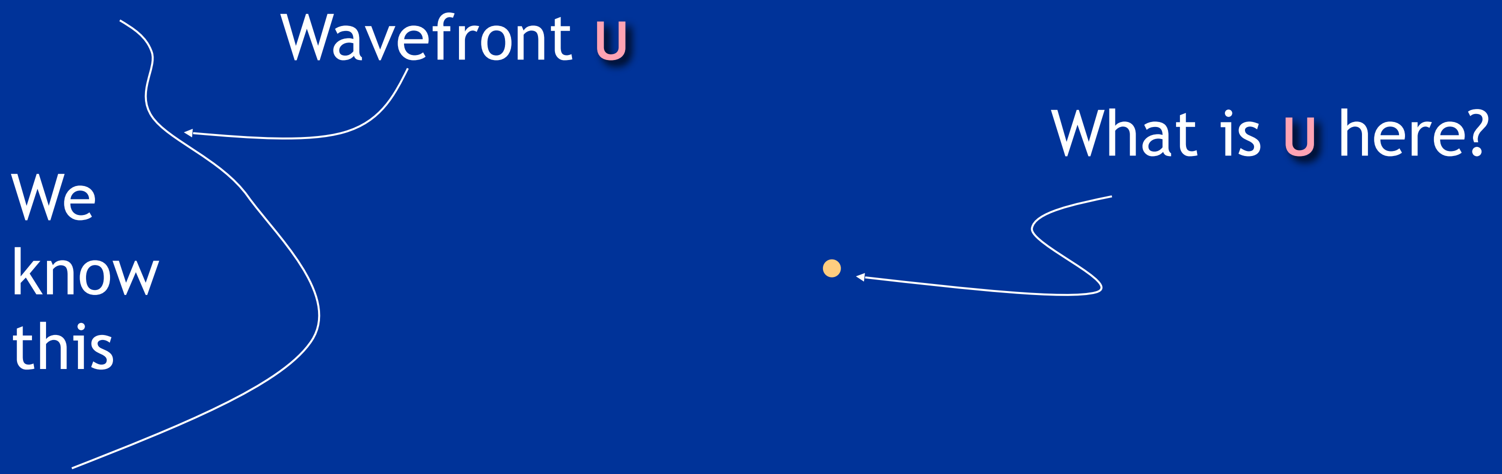


Light that has passed thru aperture, seen on screen downstream

In diffraction, apertures of an optical system limit the spatial extent of the wavefront

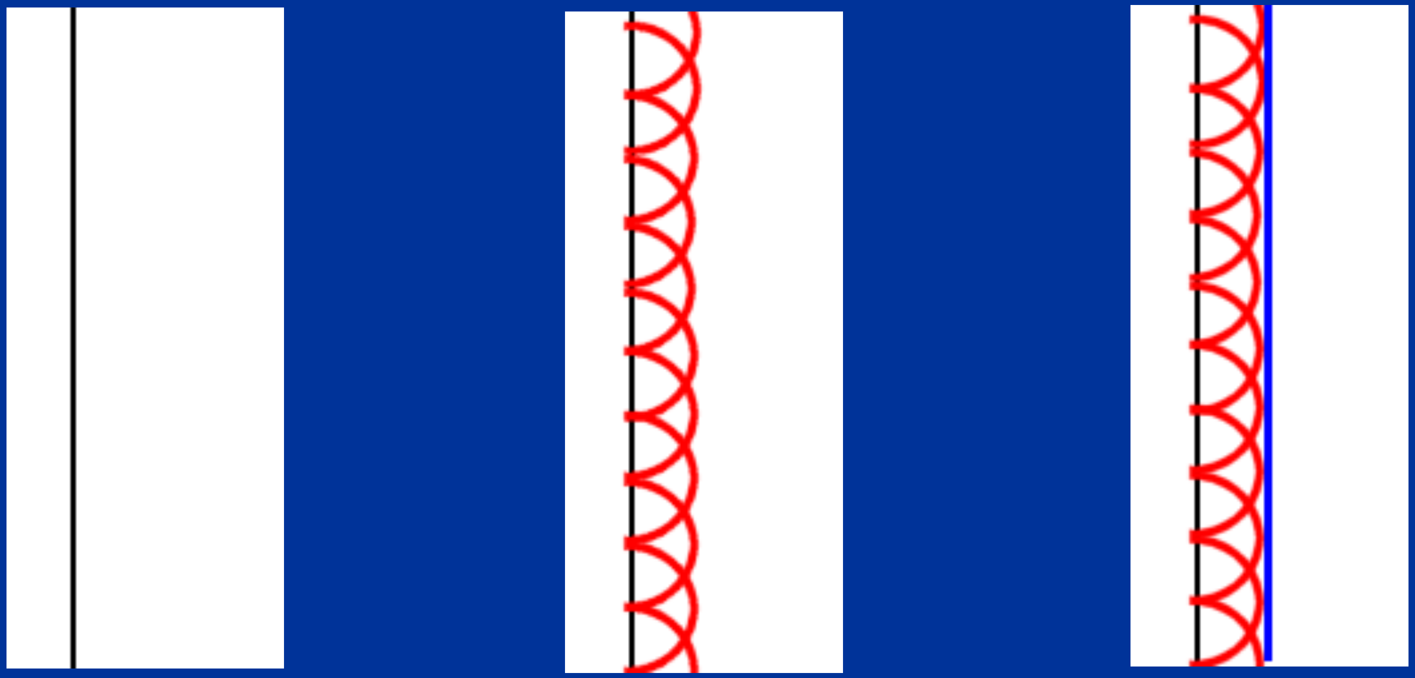
Credit: James E. Harvey, Univ. Central Florida

Diffraction Theory



Diffraction as one consequence of Huygens' Wavelets: Part 1

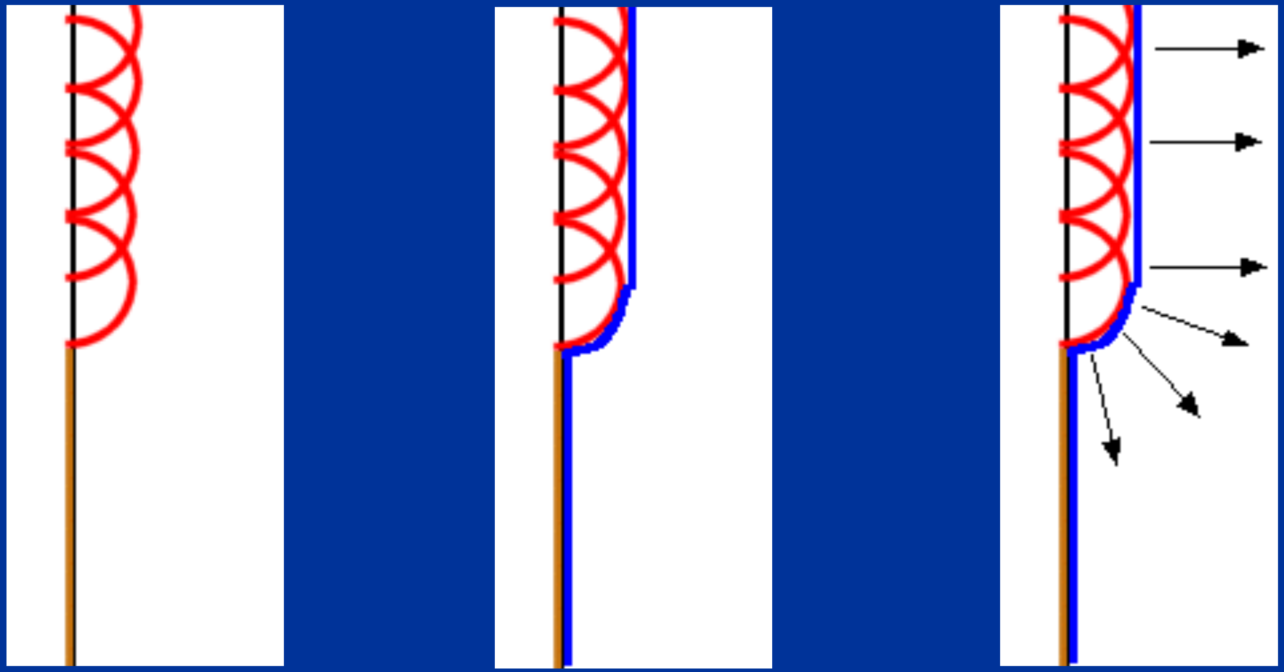
Every point on a wave front acts as a source of tiny wavelets that move forward.



Huygens' wavelets for an infinite plane wave

Diffraction as one consequence of Huygens' Wavelets: Part 2

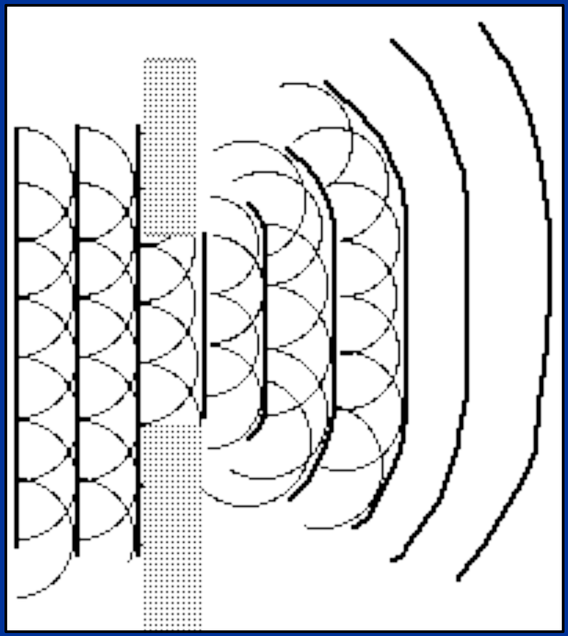
Every point on a wave front acts as a source of tiny wavelets that move forward.



Huygens' wavelets when part of a plane wave is blocked

Diffraction as one consequence of Huygens' Wavelets: Part 3

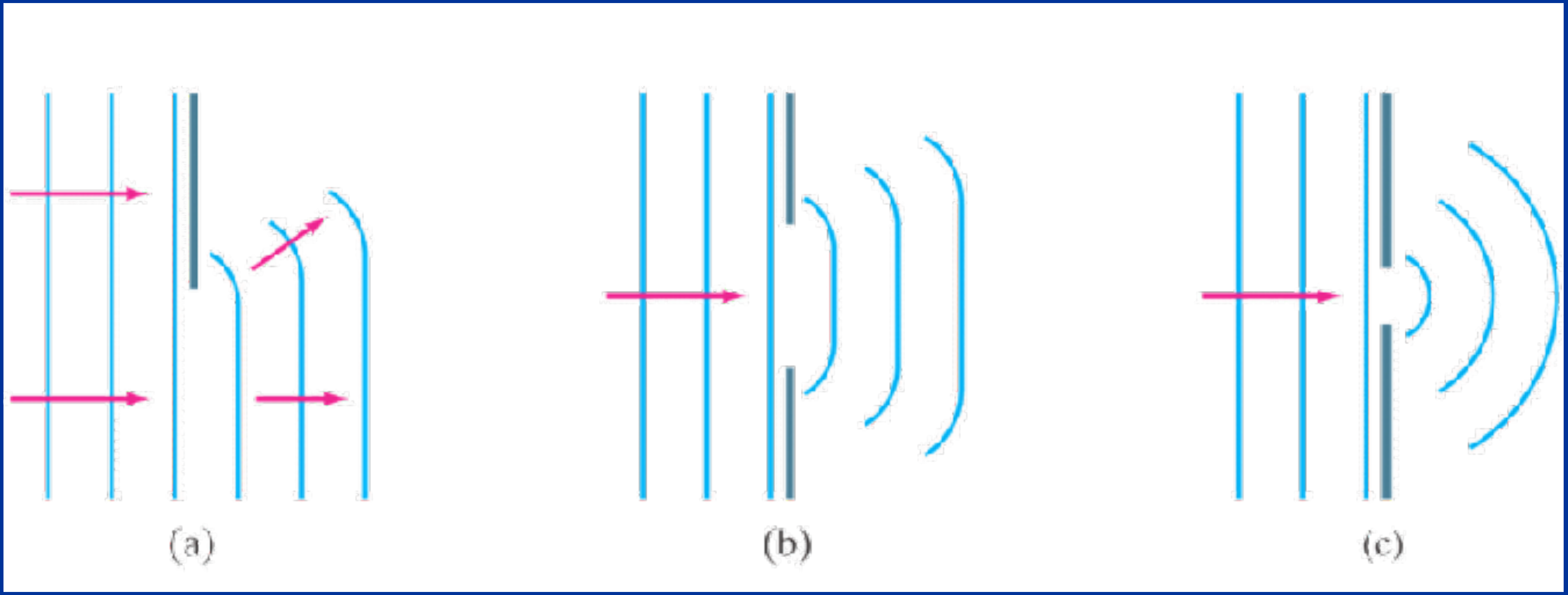
Every point on a wave front acts as a source of tiny wavelets that move forward.



Huygens' wavelets for a slit

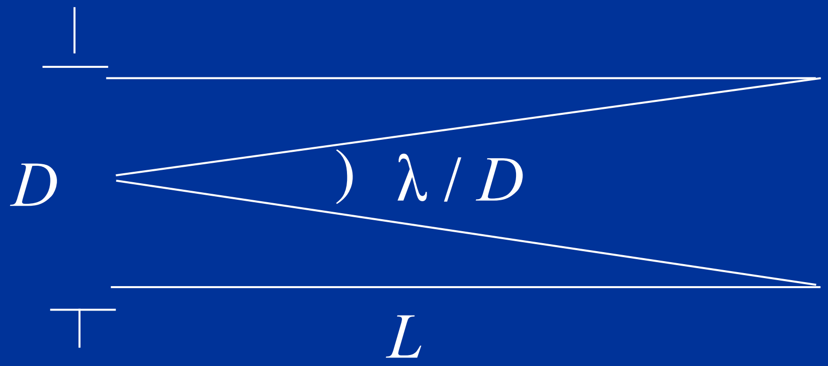
From Don Gavel

The size of the slit (relative to a wavelenth) matters



Rayleigh range

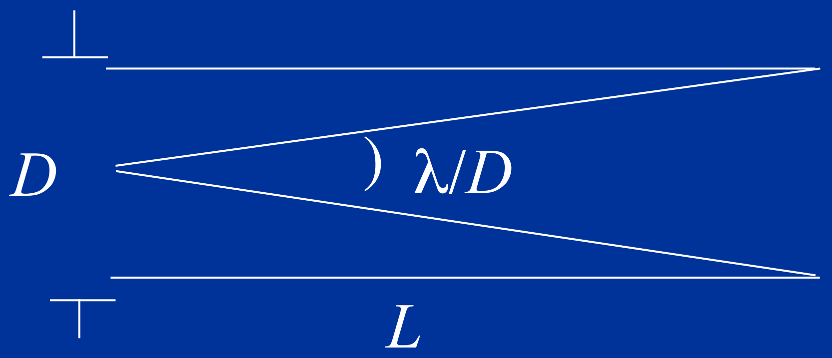
- Distance where diffraction overcomes paraxial beam propagation



$$\frac{L\lambda}{D} = D \Rightarrow L = \frac{D^2}{\lambda}$$

Fresnel Number

- Number of Fresnel zones across the beam diameter

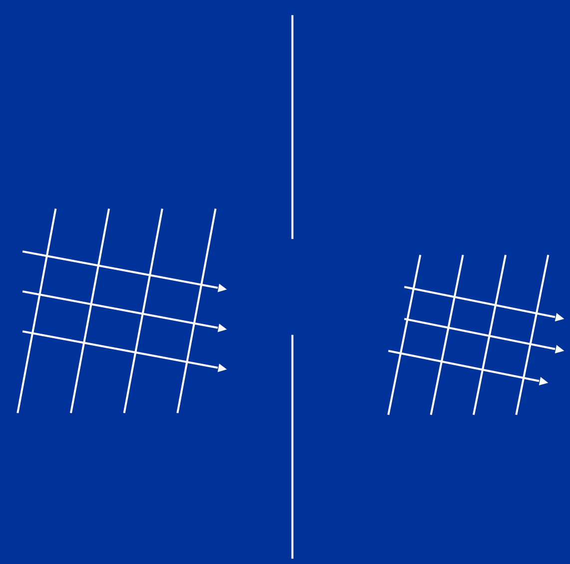


$$N = \frac{D}{L\lambda/D} = \frac{D^2}{L\lambda}$$

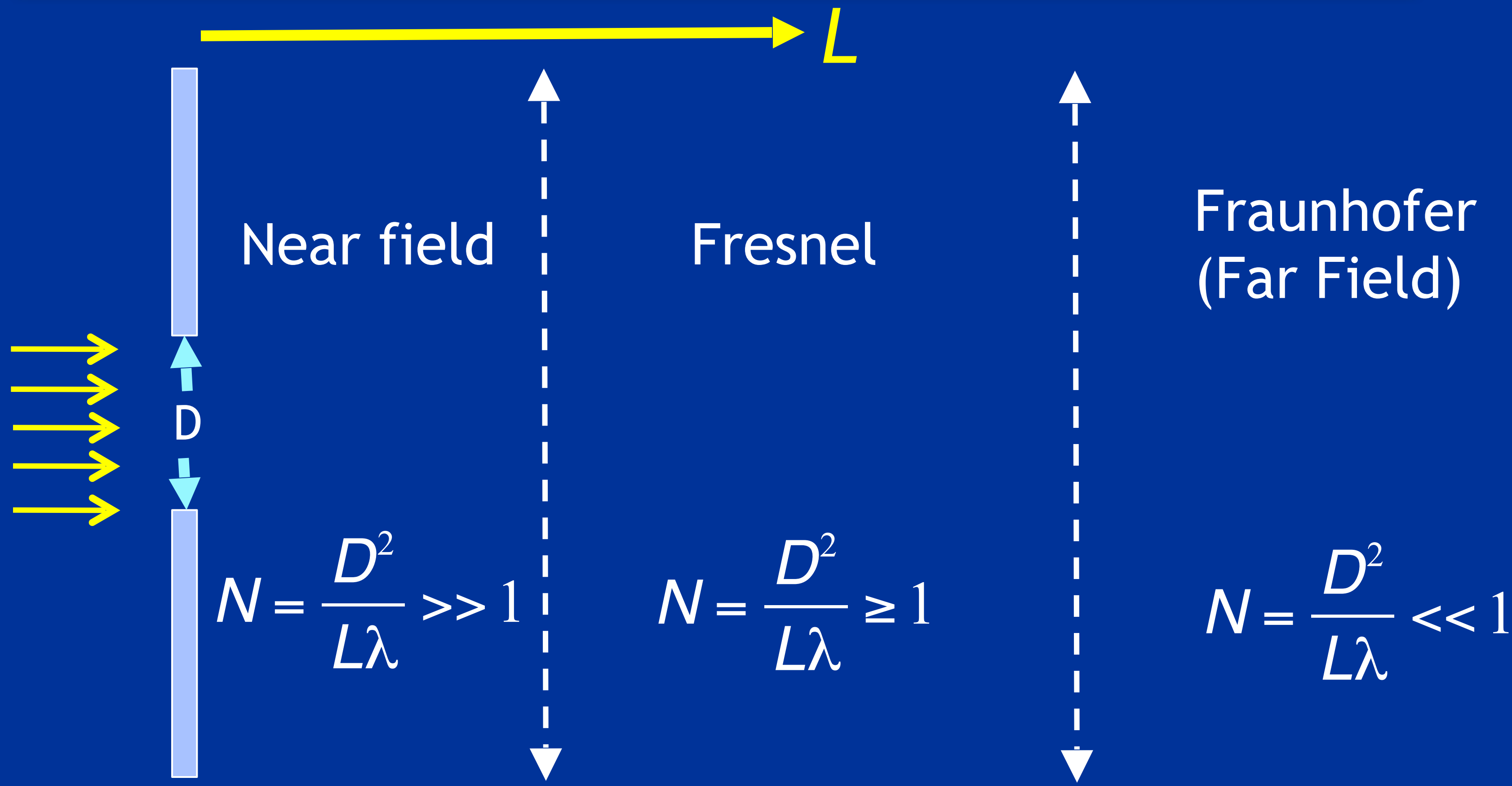
Fresnel vs. Fraunhofer diffraction

- Fresnel regime is the near-field regime: the wave fronts are curved, and their mathematical description is more involved.

- Very far from a point source, wavefronts almost plane waves.
- Fraunhofer approximation valid when source, aperture, and detector are all very far apart (or when lenses are used to convert spherical waves into plane waves)



Regions of validity for diffraction calculations



$$N = \frac{D^2}{L\lambda} >> 1$$

$$N = \frac{D^2}{L\lambda} \geq 1$$

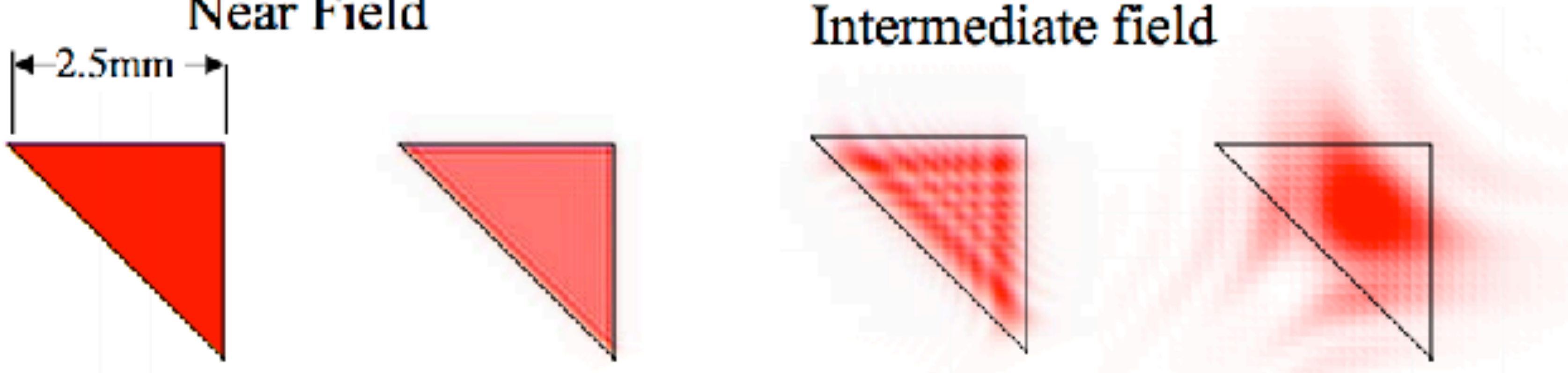
$$N = \frac{D^2}{L\lambda} << 1$$

The farther you are from the slit, the easier it is to calculate the diffraction pattern

Pattern on screen at various distances

Near Field

Intermediate field

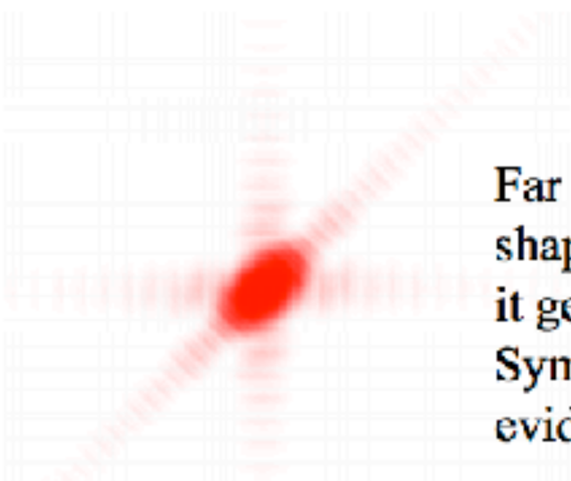


Immediately behind screen

25 mm from screen, bright fringes just inside edges

250 mm light penetrates into shadow region

2500 mm pattern doesn’t closely resemble mask



Far field – at a large enough distance shape of pattern no longer changes but it gets bigger with larger distance. Symmetry of original mask still is evident.

Credit: Bill Molander, LLNL

Fraunhofer diffraction equation

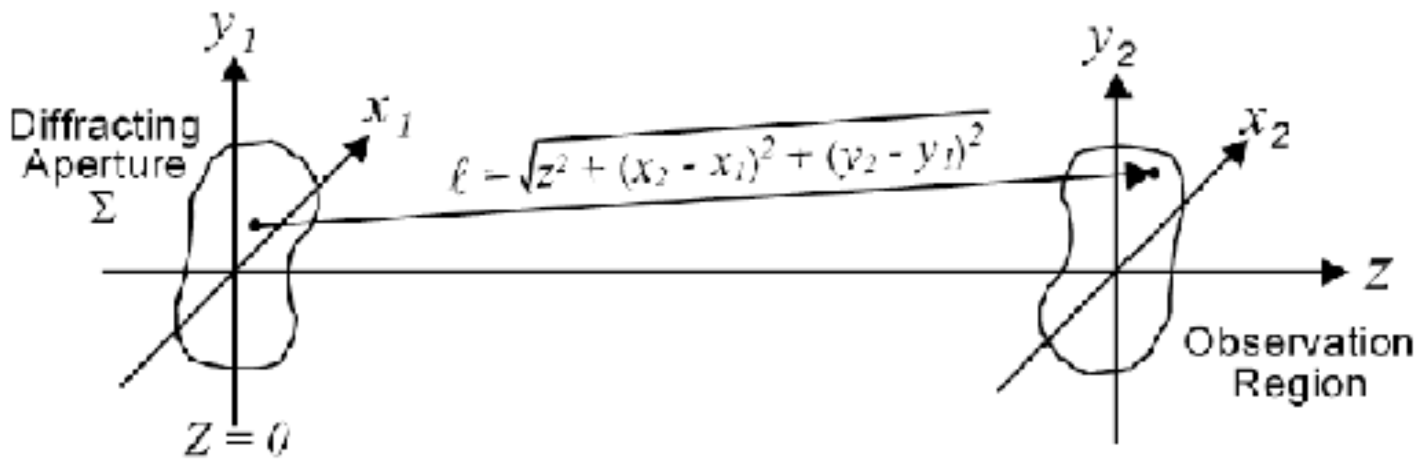


Figure 3-6. Geometrical relationship between diffracting aperture and observation space.

$$U_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x_2^2 + y_2^2)\right] \mathcal{F}\{U_1(x_1, y_1)\}\Big|_{\substack{\xi = x_2/\lambda z \\ \eta = y_2/\lambda z}}$$

F is Fourier Transform

Please note that Fourier transforming a function of x and y results in a function of spatial frequencies ξ and η, which must then be evaluated at $\xi = x_2 / \lambda z$ and $\eta = y_2 / \lambda z$.

Fraunhofer diffraction, continued

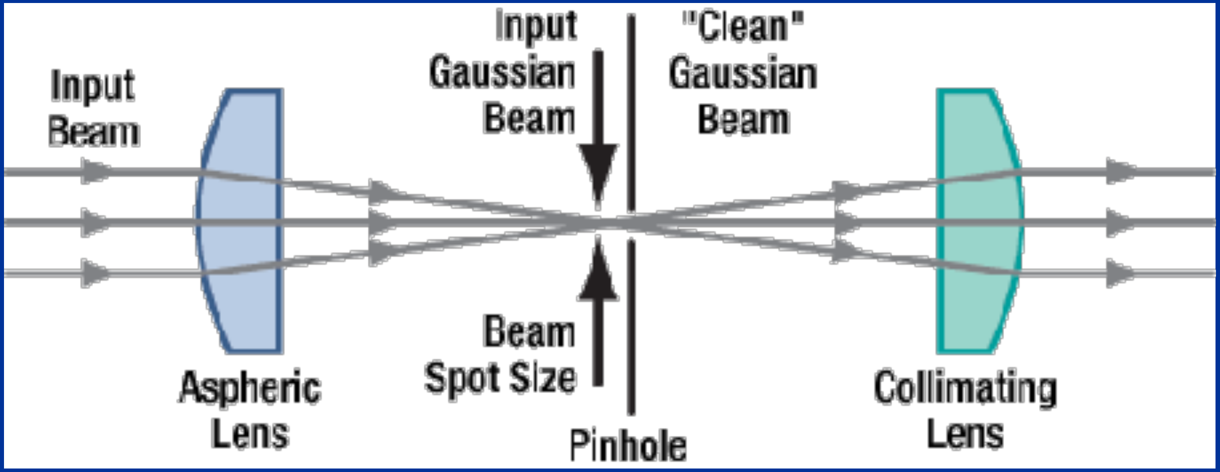
$$U_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x_2^2 + y_2^2)\right] \mathcal{F}\{U_1(x_1, y_1)\}\Big|_{\substack{\xi = x_2/\lambda z \\ \eta = y_2/\lambda z}}$$

F is Fourier Transform

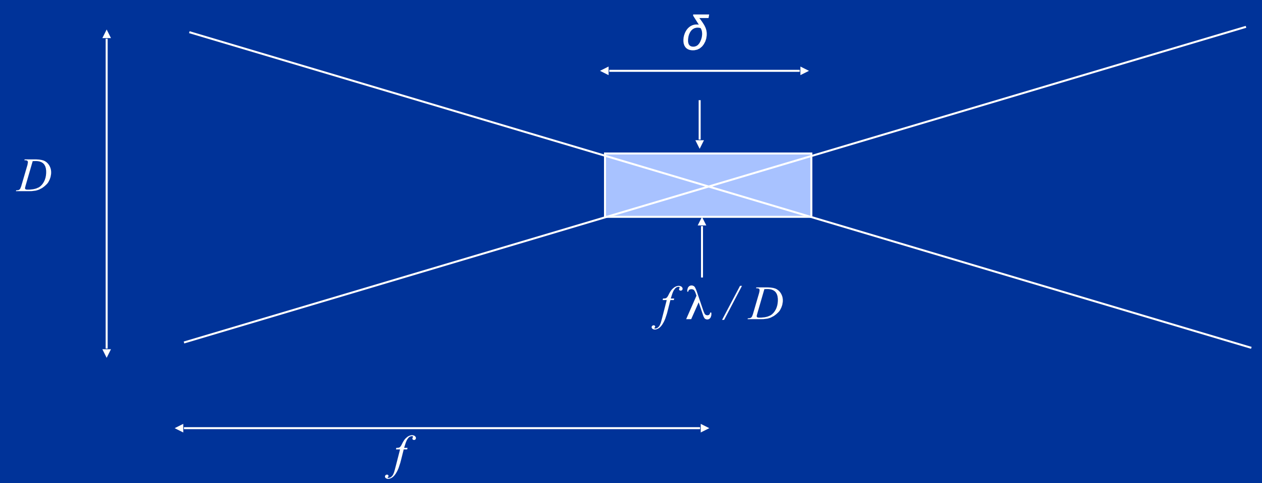
- In the "far field" (Fraunhofer limit) the diffracted field U_2 can be computed from the incident field U_1 by a phase factor times the Fourier transform of U_1
- "Image plane is Fourier transform of pupil plane"

Image plane is Fourier transform of pupil plane

- Leads to principle of a "spatial filter"
- Say you have a beam with too many intensity fluctuations on small spatial scales
  - Small spatial scales = high spatial frequencies
- If you focus the beam through a small pinhole, the high spatial frequencies will be focused at larger distances from the axis, and will be blocked by the pinhole



Depth of focus

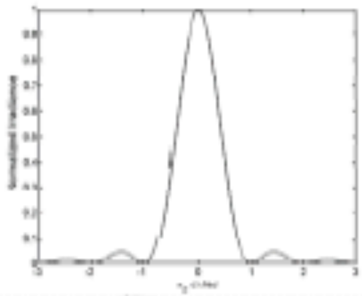


$$\frac{f}{D} = \frac{\delta/2}{f\lambda/D} \Rightarrow \delta = 2\left(\frac{f}{D}\right)^2 \lambda$$

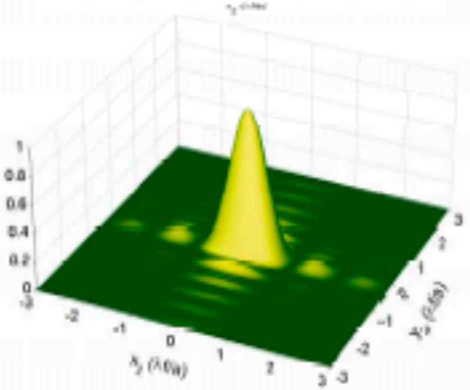
Rectangular Aperture

$$E_2(x_2, y_2) = \frac{E_o\, a^2 d^2}{\lambda^2 f^2}\,\mathrm{sinc}^2\!\left(\frac{x_2}{\lambda f/a}, \frac{y_2}{\lambda f/d}\right)$$

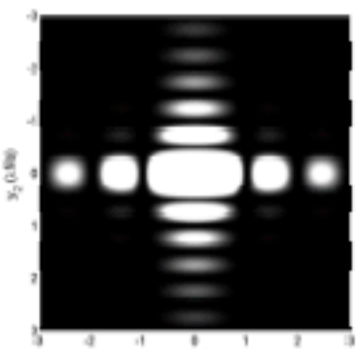
a.)



b.



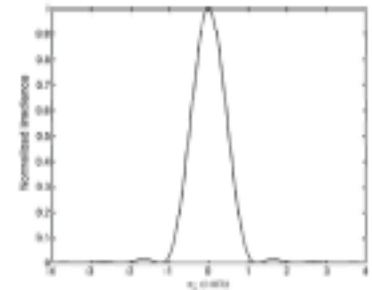
c.)



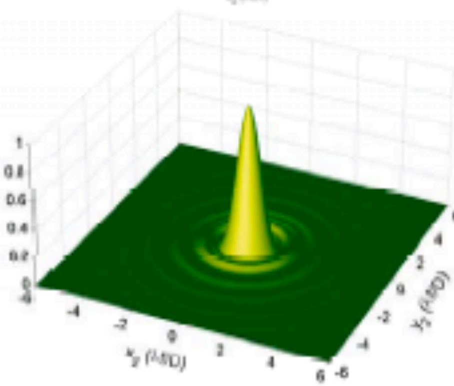
Circular Aperture

$$E_2(x_2, y_2) = E_o\left(\frac{\pi D^2}{4\lambda f}\right)^2 \mathrm{somb}^2\!\left(\frac{r_2}{\lambda f/D}\right)$$

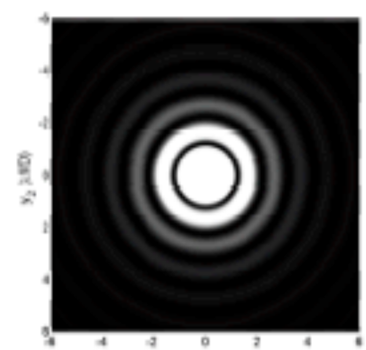
a.)



b



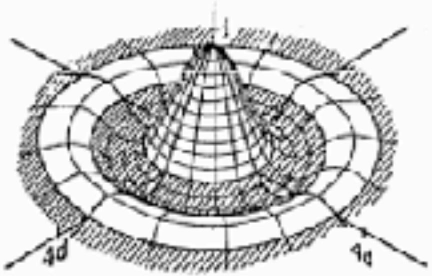
c.)

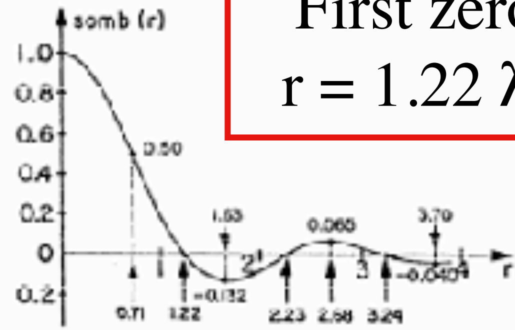


Details of diffraction from circular aperture

1) Amplitude

$$\mathrm{somb}(r) = 2\,J_1(\pi r)/(\pi r)$$

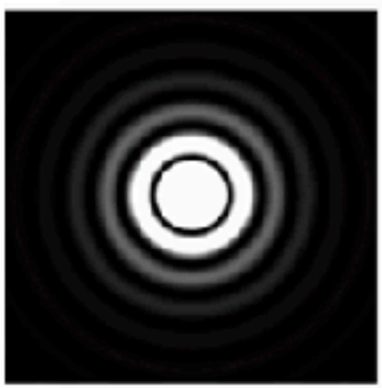




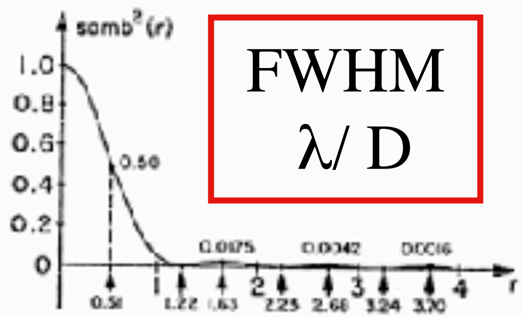
First zero at
r = 1.22 λ/ D

2) Intensity

Airy Pattern

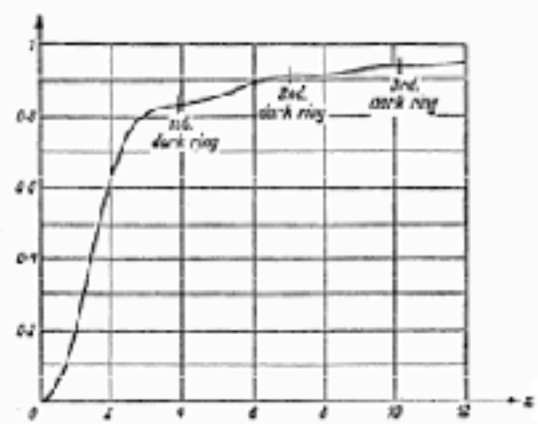


$\mathrm{somb}^2(r)$



FWHM
λ/ D

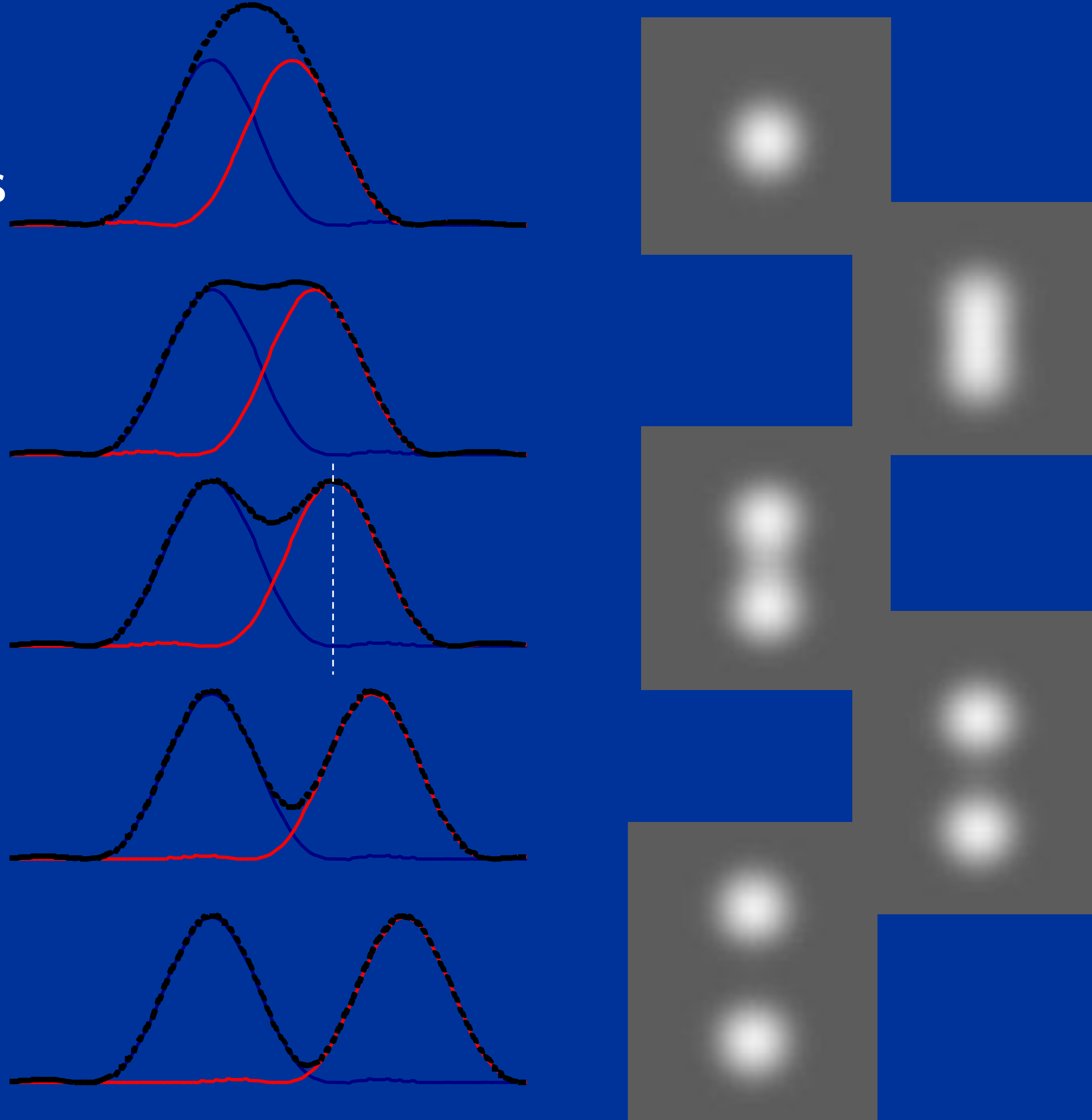
Fractional Encircled Energy



2 unresolved point sources

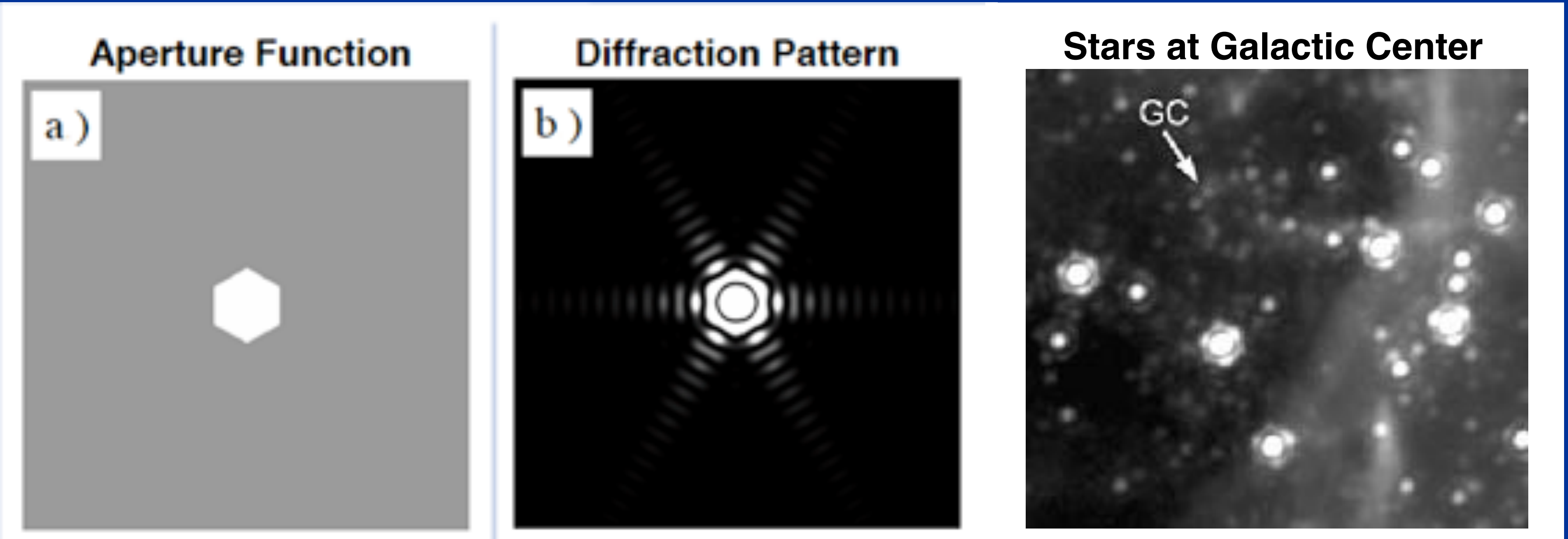
Rayleigh resolution limit: $\Theta = 1.22\ \lambda/D$

Resolved



Credit: Austin Roorda

Diffraction pattern from hexagonal Keck telescope



Ghez: Keck laser guide star AO

Takeaways

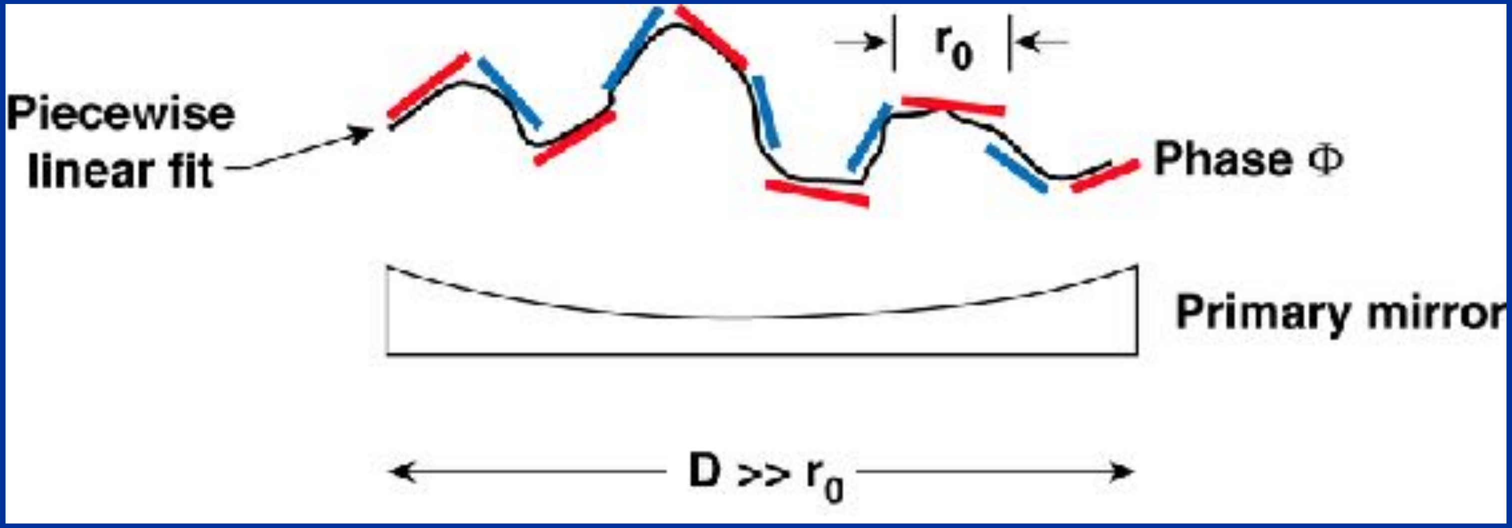
- Light behavior is modeled well as a wave phenomena (Huygens, Maxwell)
- Description of diffraction depends on how far you are from the source (Fresnel, Fraunhofer)
- Geometric and diffractive phenomena seen in the lab (Rayleigh range, diffraction limit, depth of focus…)
- Image formation with wave optics

Deformable Mirrors

Outline of Deformable Mirror Lecture

- Performance requirements for wavefront correction
- Types of deformable mirrors
  - Actuator types
  - Segmented DMs
  - Continuous face-sheet DMs
  - Bimorph DMs
  - Adaptive Secondary mirrors
  - MEMS DMs
  - (Liquid crystal devices)
- Summary: fitting error, what does the future hold?

Deformable mirror requirements: r_0 sets number of degrees of freedom of an AO system



- Divide primary mirror into "subapertures" of diameter r_0
- Number of subapertures ~ $(D / r_0)^2$ where r_0 is evaluated at the desired observing wavelength

Overview of wavefront correction

- Divide pupil into regions of ~ size r_0 , do "best fit" to wavefront. Diameter of subaperture = d
- Several types of deformable mirror (DM), each has its own characteristic "fitting error"

$$\sigma_{fitting}^2 = \mu \, (d \, / \, r_0)^{5/3} \text{ rad}^2$$

- Exactly how large d is relative to r_0 is a design decision; depends on overall error budget

DM requirements (1)

- Dynamic range: stroke (total up and down range)
  - Typical "stroke" for astronomy depends on telescope diameter:
    - ± several microns for 10 m telescope
    - ± 10-15 microns for 30 m telescope
  - Question: Why bigger for larger telescopes?

- Temporal frequency response:
  - DM must respond faster than a fraction of the coherence time τ_0

- Influence function of actuators:
  - Shape of mirror surface when you push just one actuator (like a Greens' function)
  - Can optimize your AO system with a particular influence function, but performance is pretty forgiving

DM requirements (2)

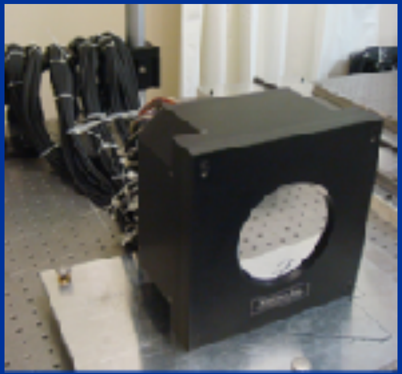
- Surface quality:
  - Small-scale bumps can't be corrected by AO
- Hysteresis of actuators:
  - Repeatability
  - Want actuators to go back to same position when you apply the same voltage
- Power dissipation:
  - Don't want too much resistive loss in actuators, because heat is bad ("seeing", distorts mirror)
  - Lower voltage is better (easier to use, less power dissipation)
- DM size:
  - Not so critical for current telescope diameters
  - For 30-m telescope need big DMs: at least 30 cm across
    - Consequence of the Lagrange invariant

$$y_1 \vartheta_1 = y_2 \vartheta_2$$

Types of deformable mirrors: conventional (large)

- Segmented
  - Made of separate segments with small gaps

- "Continuous face-sheet"
  - Thin glass sheet with actuators glued to the back



- Bimorph
  - 2 piezoelectric wafers bonded together with array of electrodes between them. Front surface acts as mirror.

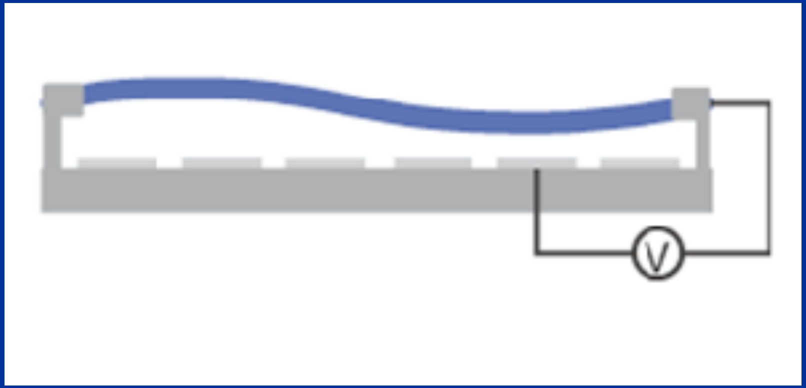
Types of deformable mirrors: small and/or unconventional (1)

- Liquid crystal spatial light modulators
  - Technology similar to LCDs
  - Applied voltage orients long thin molecules, changes *n*
  - Not practical for astronomy

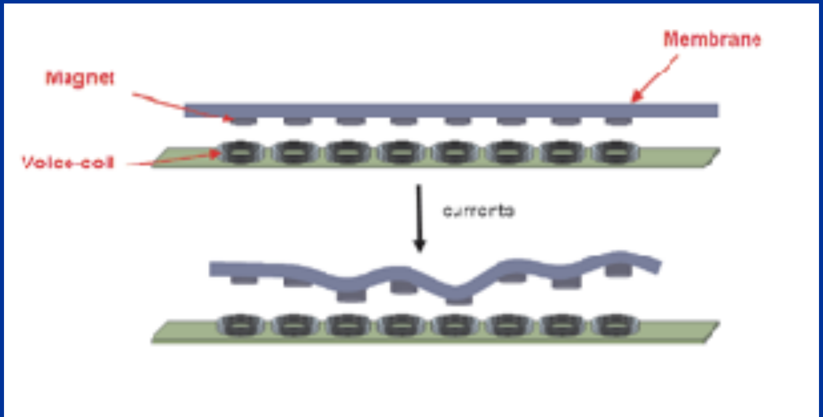
- MEMS (micro-electro-mechanical systems)
  - Fabricated using micro-fabrication methods of integrated circuit industry
  - Potential to be inexpensive

Types of deformable mirrors: small and/or unconventional (2)

- Membrane mirrors
  - Low order correction
  - Example: OKO (Flexible Optical BV)

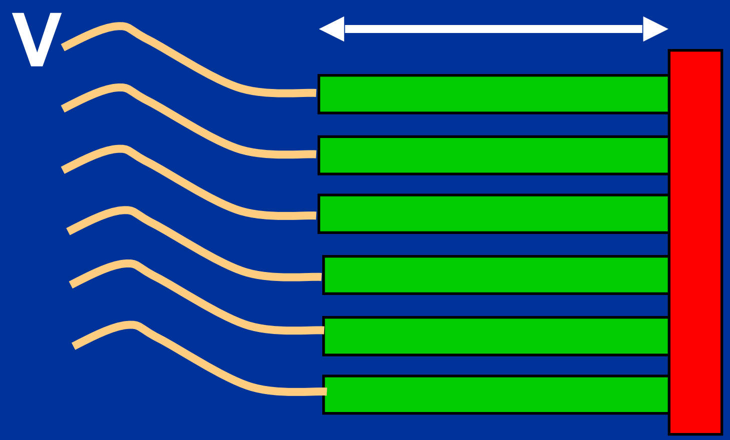


- Magnetically actuated mirrors
  - High stroke, high bandwidth
  - Example: ALPAO



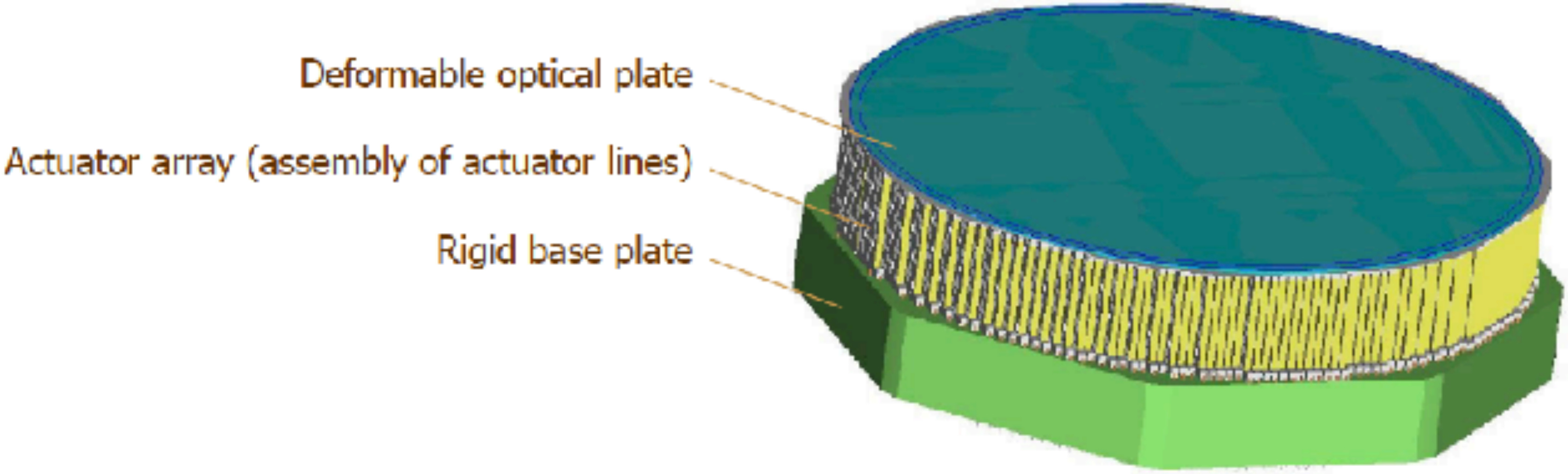
Typical role of actuators in a conventional continuous face-sheet DM

- Actuators are glued to back of thin glass sheet (has a reflective coating on the front)
- When you apply a voltage to the actuator (PZT, PMN), it expands or contracts in length, thereby pushing or pulling on the mirror



Example from CILAS

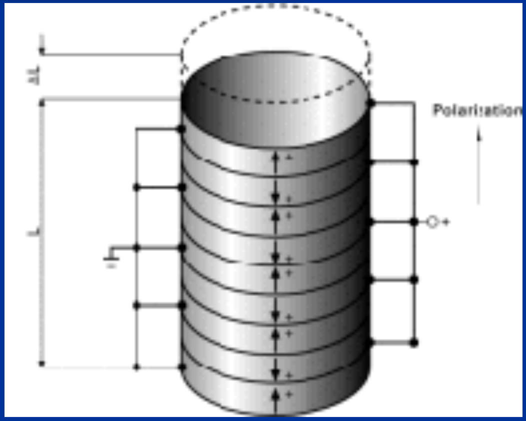
General design of DMs

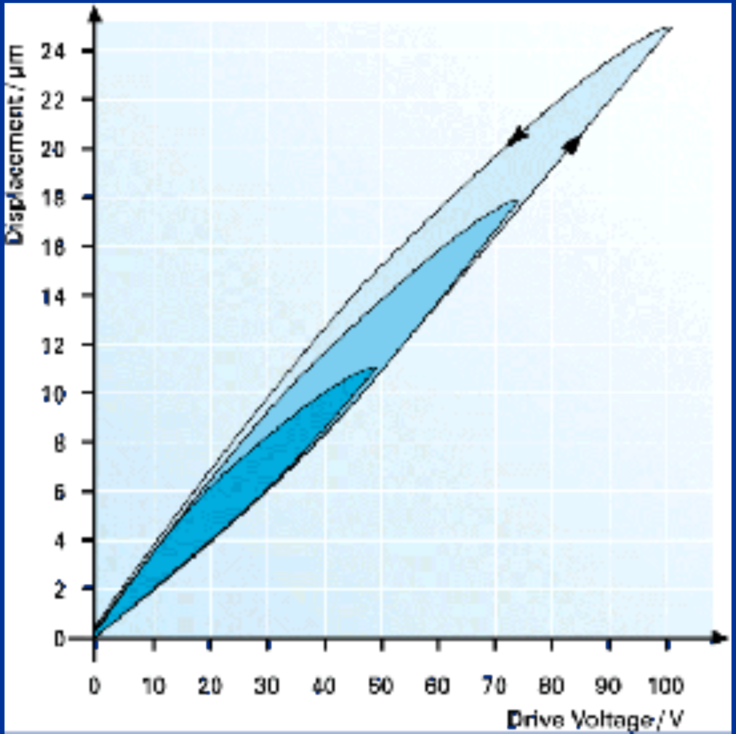


Example (design for TMT): ~ 4500 actuators, 5 mm spacing, Ø ~ 400 mm

Types of actuator: Piezoelectric

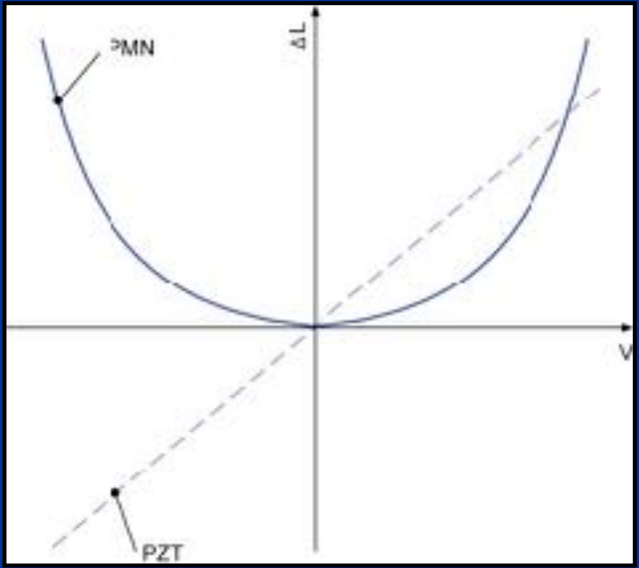
- Piezo from Greek for Pressure
- PZT (lead zirconate titanate) gets longer or shorter when you apply V
- Stack of PZT ceramic disks with integral electrodes
- Displacement linear in voltage
- Typically 150 Volts
  ⇒ Δx ~ 10 microns
- 10-20% hysteresis
  (actuator doesn't go back to exactly where it started)





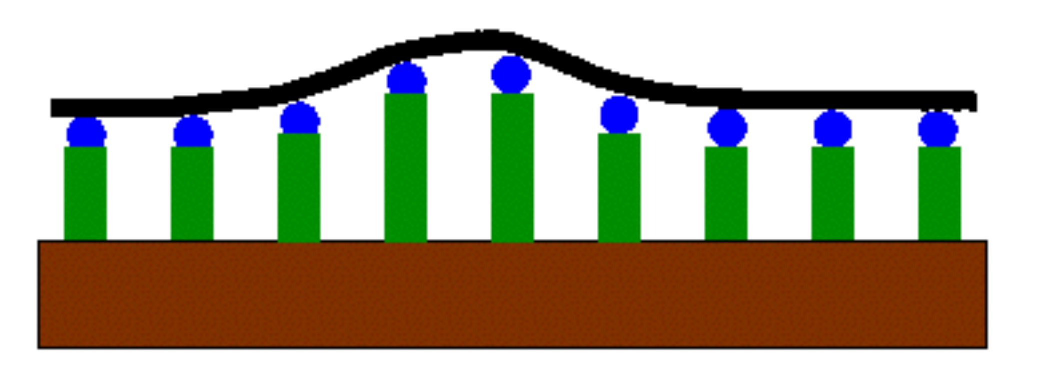
Types of actuator: PMN

- Lead magnesium niobate (PMN)
- Electrostrictive:
  - Material gets longer in response to an applied electric field
- Quadratic response (non-linear)
- Can "push" and "pull" if a bias is applied
- Hysteresis can be lower than PZT in some temperature ranges
- Both displacement and hysteresis depend on temperature (PMN is more temperature sensitive than PZT)



Good reference (figures on these slides): www.physikinstrumente.com/en/products/piezo_tutorial.php

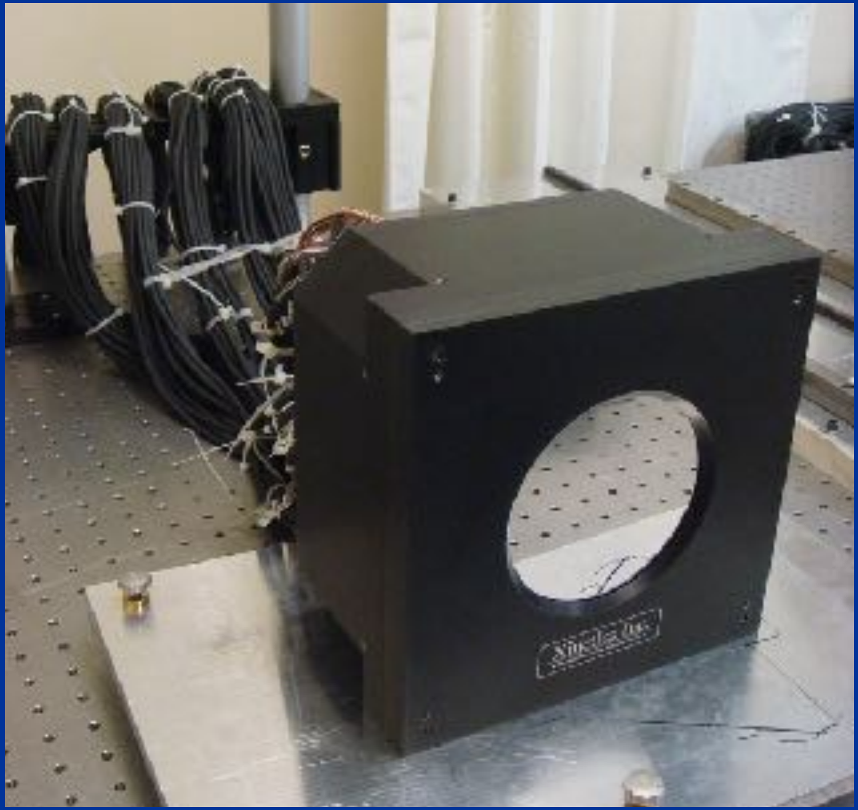
Continuous face-sheet DMs: Design considerations

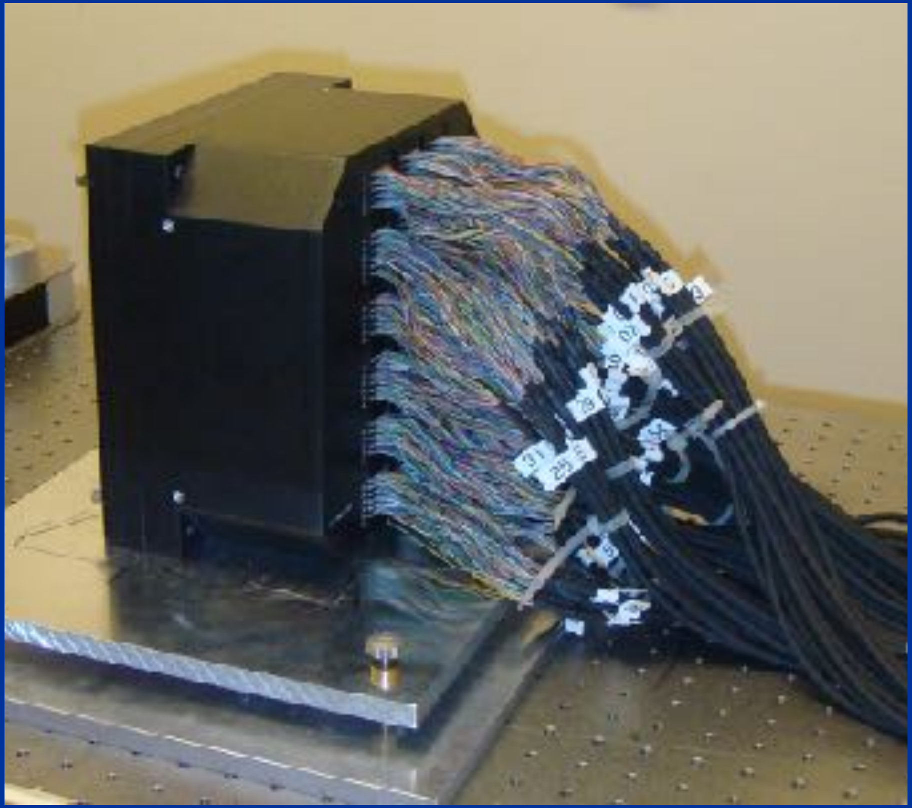


- Facesheet thickness must be large enough to maintain flatness during polishing, but thin enough to deflect when pushed or pulled by actuators
- Thickness also determines "influence function"
  - Response of mirror shape to "push" by 1 actuator
  - Thick face sheets ⇒ broad influence function
  - Thin face sheets ⇒ more peaked influence function
- Actuators have to be stiff, so they won't bend sideways

Palm 3000 High-Order Deformable Mirror: 4356 actuators!

Credit: A. Bouchez

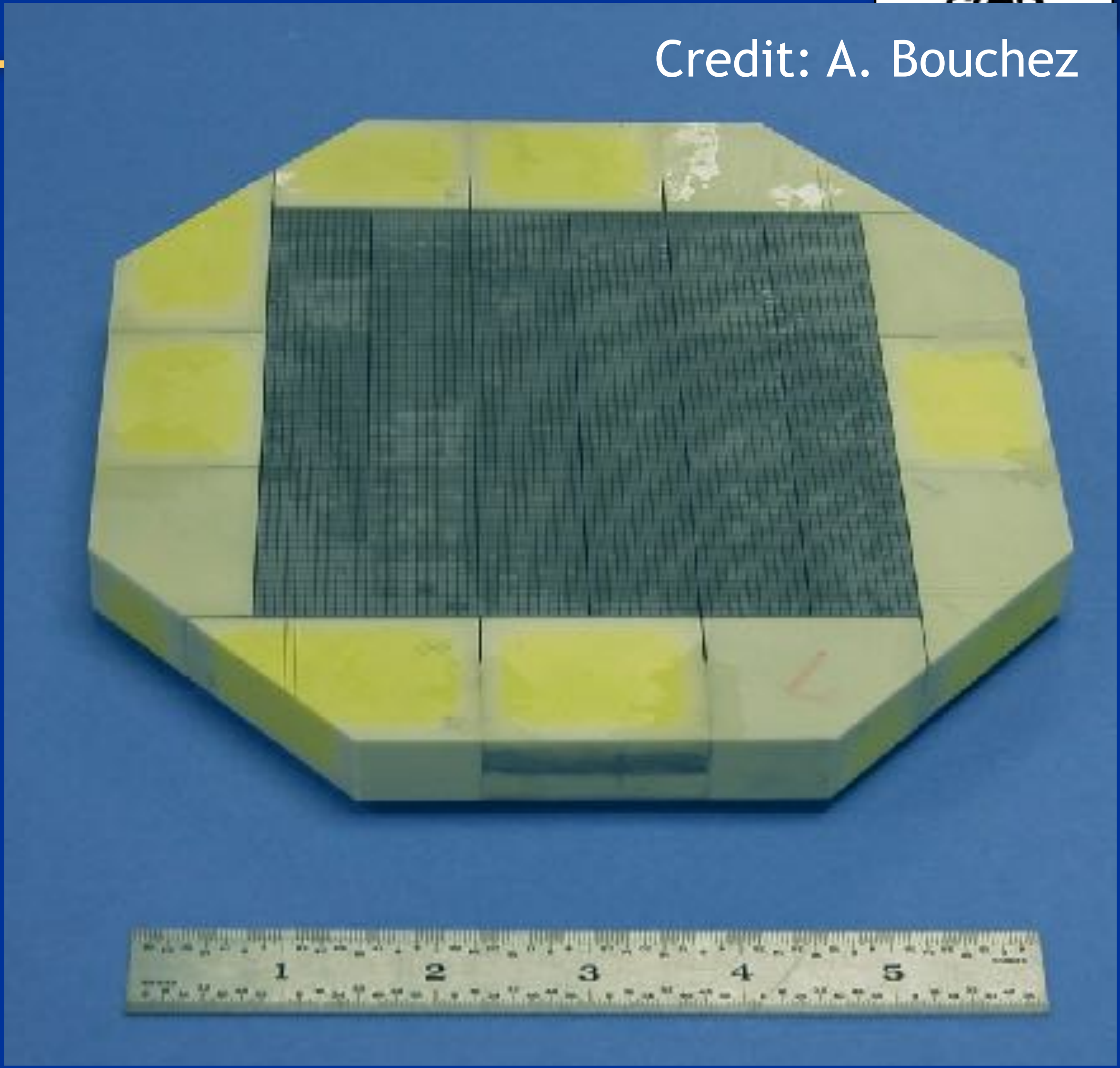




Xinetics Inc. for Mt. Palomar “Palm 3000” AO system

Palm 3000 DM Actuator Structure

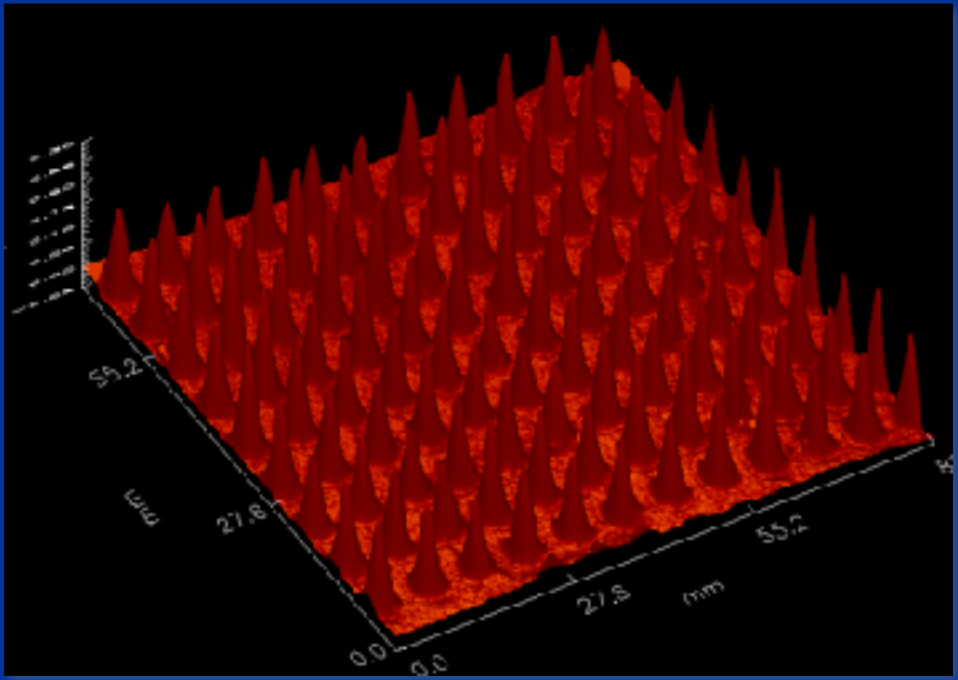
- Actuators machined from monolithic blocks of PMN
- 6x6 mosaic of 11x11 actuator blocks
- 2mm thick Zerodur glass facesheet
- Stroke ~1.4 μm without face sheet, uniform to 9% RMS.



Prior to face sheet bonding

Palm 3000 DM: Influence Functions

Credit: A. Bouchez



- Influence function: response to one actuator
- Zygo interferometer surface map of a portion of the mirror, with every 4th actuator poked

Bimorph mirrors are well matched to curvature sensing AO systems

- Electrode pattern shaped to match sub-apertures in curvature sensor
- Mirror shape W(x,y) obeys Poisson Equation

$$\nabla^2 \left(\nabla^2 W + AV \right) = 0$$

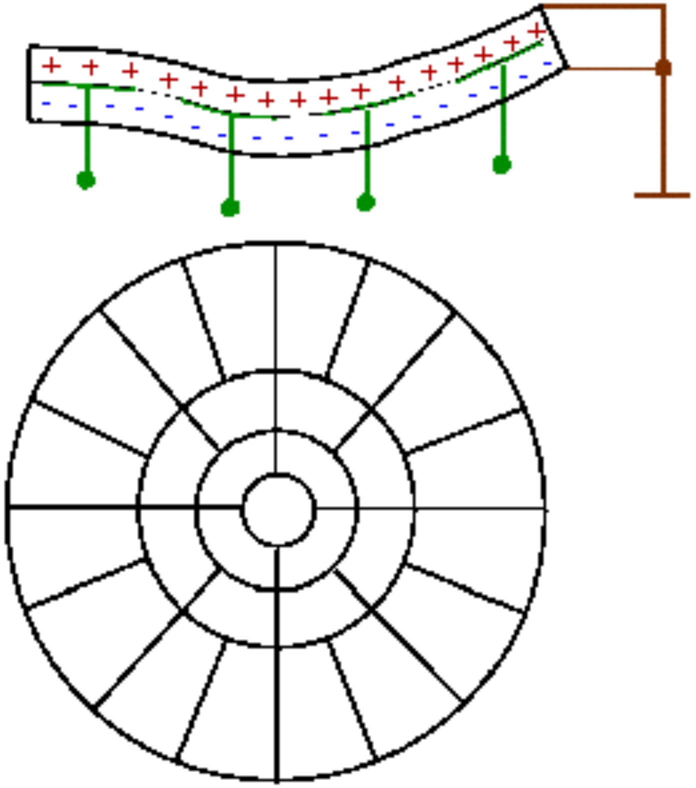
where $A = 8 d_{31} / t^2$

d_{31} is the transverse piezo constant

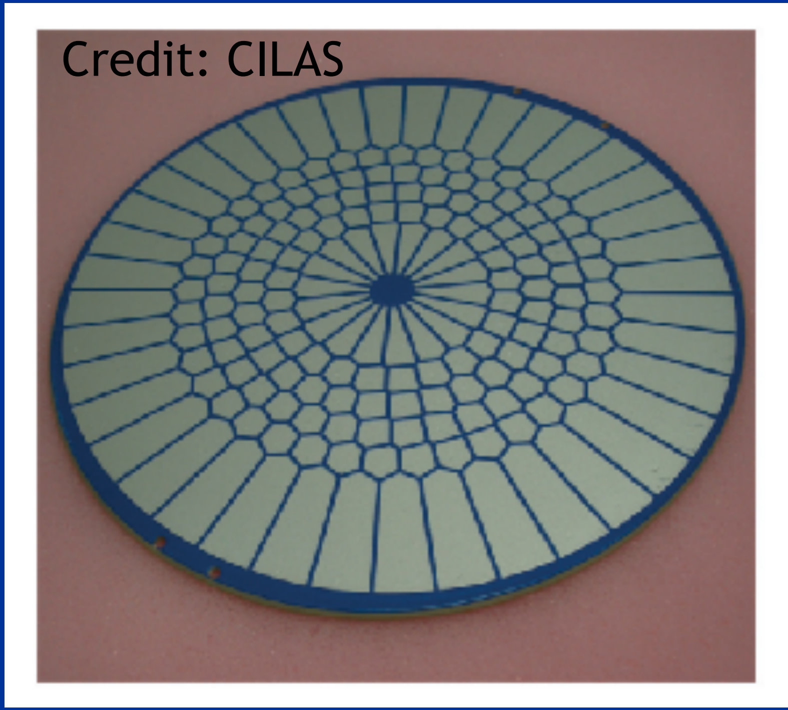
t is the thickness

$V(x,y)$ is the voltage distribution

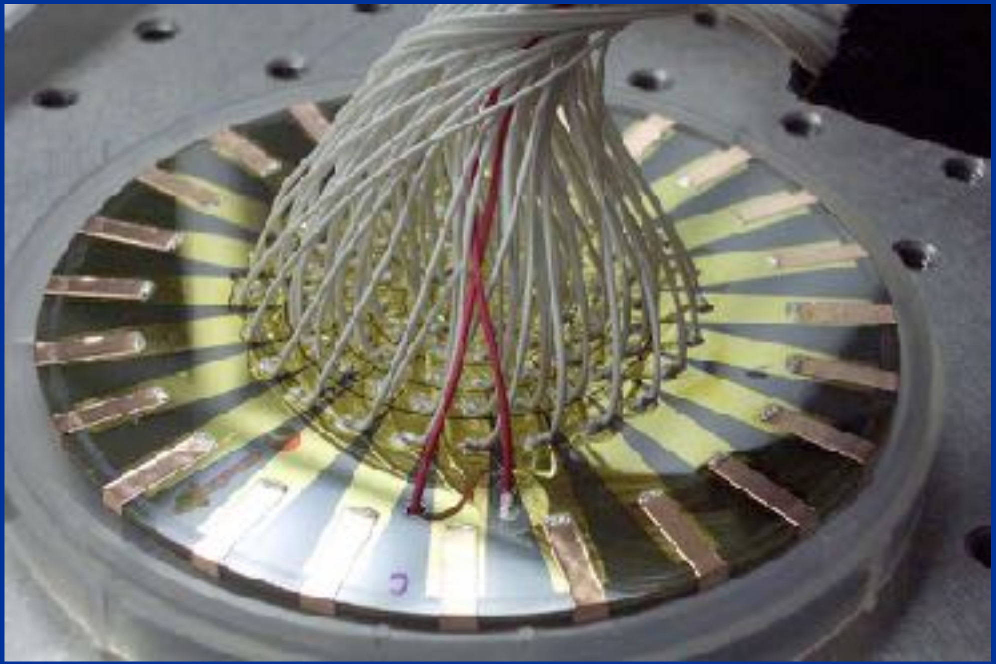
Credit: A. Tokovinin



Bimorph deformable mirrors: embedded electrodes



Electrode Pattern



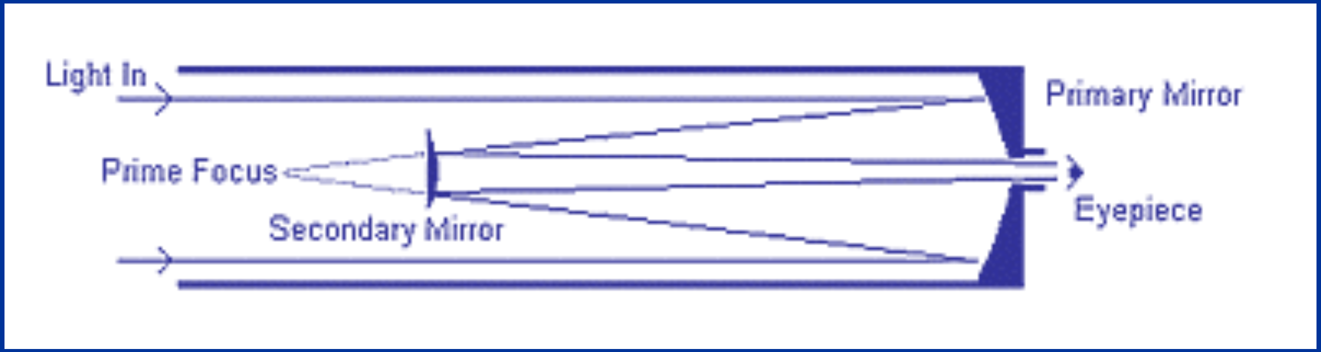
Wiring on back

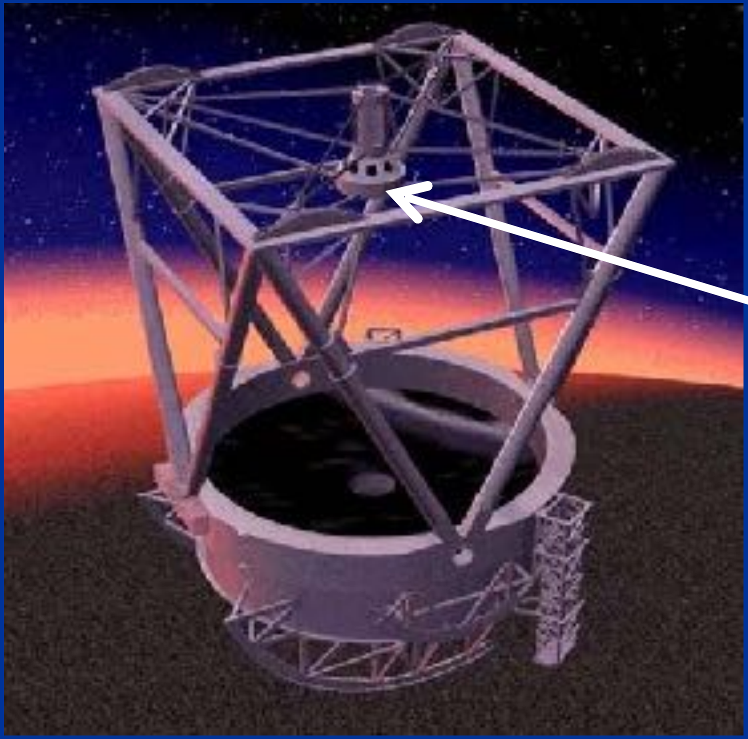
- ESO's Multi Application Curvature Adaptive Optics (MACAO) system uses a 60-element bimorph DM and a 60-element curvature wavefront sensor
- Very successful: used for interferometry of the four 8-m telescopes

Deformable Secondary Mirrors

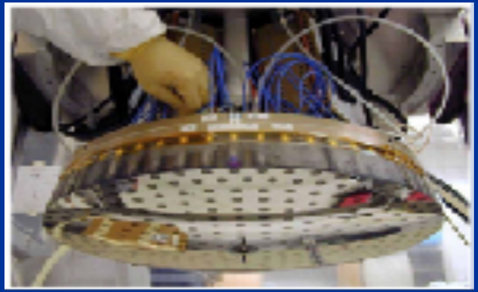
- Pioneered by U. Arizona and Arcetri Observatory in Italy
- Developed further by Microgate (Italy)
- First Generation: MMT 336 actuator adaptive secondary
- Second Generation: LBT and Magellan telescope (672 actuators)
- Third Generation: VLT AO facility (1170 actuators)
- Lower Power approach being explored by UH, TNO, and here at UCSC.

Cassegrain telescope concept





Secondary mirror



General concept for adaptive secondary mirrors (Arizona, Arcetri, MicroGate)

- Voicecoil actuators are located on rigid backplate or “reference body”
- Thin shell mirror has permanent magnets glued to rear surface; these suspend the shell below the backplate
- Capacitive sensors on backplate give an independent measurement of the shell position

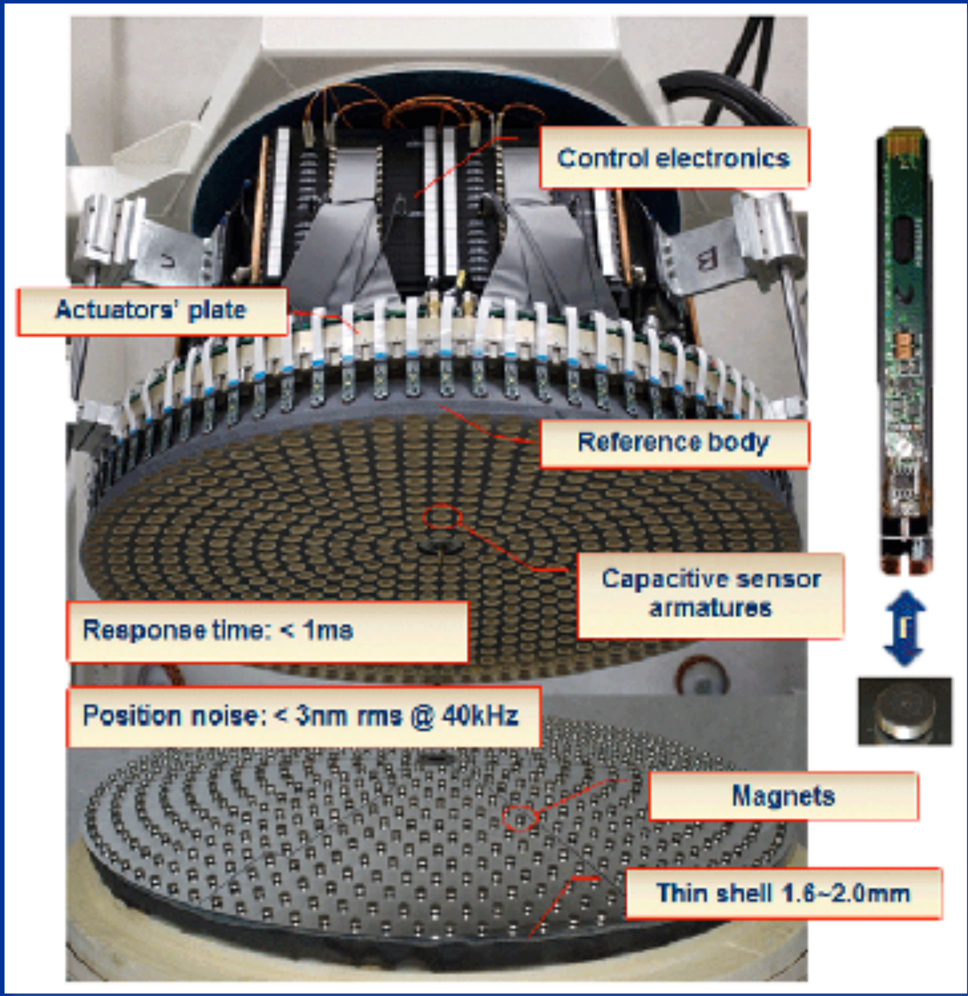


Diagram from MicroGate’s website

Shell is VERY thin!

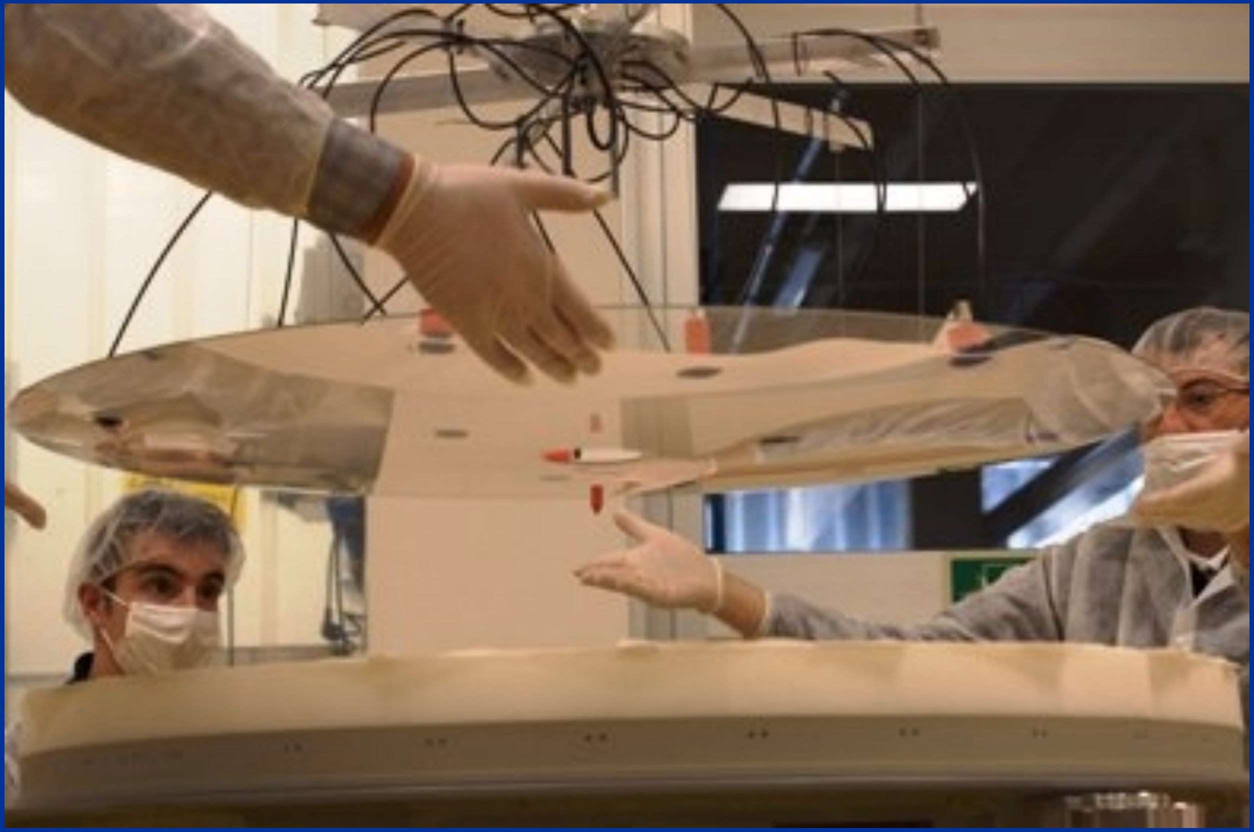
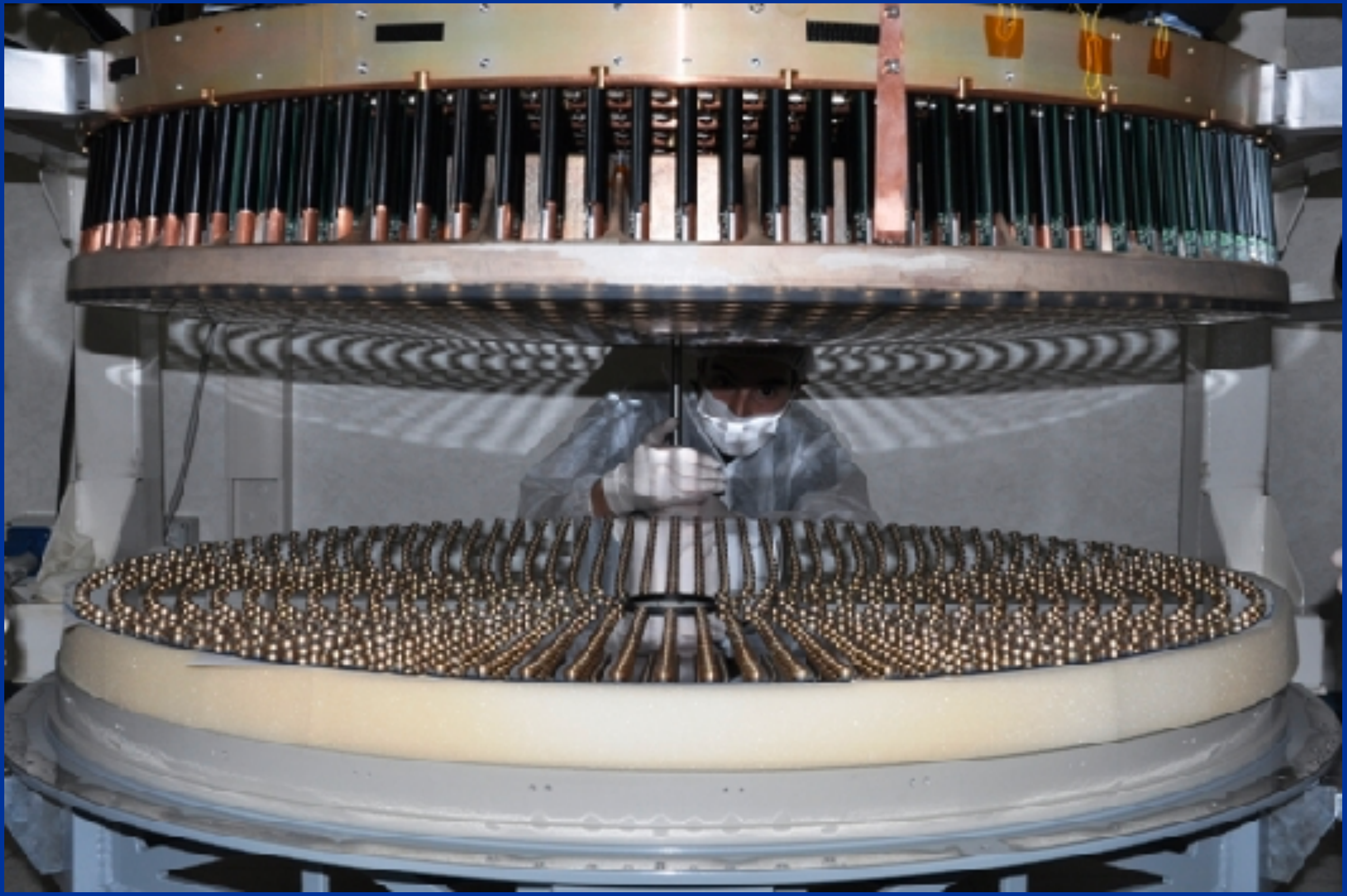


Photo Credit: ADS International

Voice-Coil Actuators viewed from the side



Deformable secondaries: embedded permanent magnets

LBT DM: magnet array

LBT DM: magnet close-up

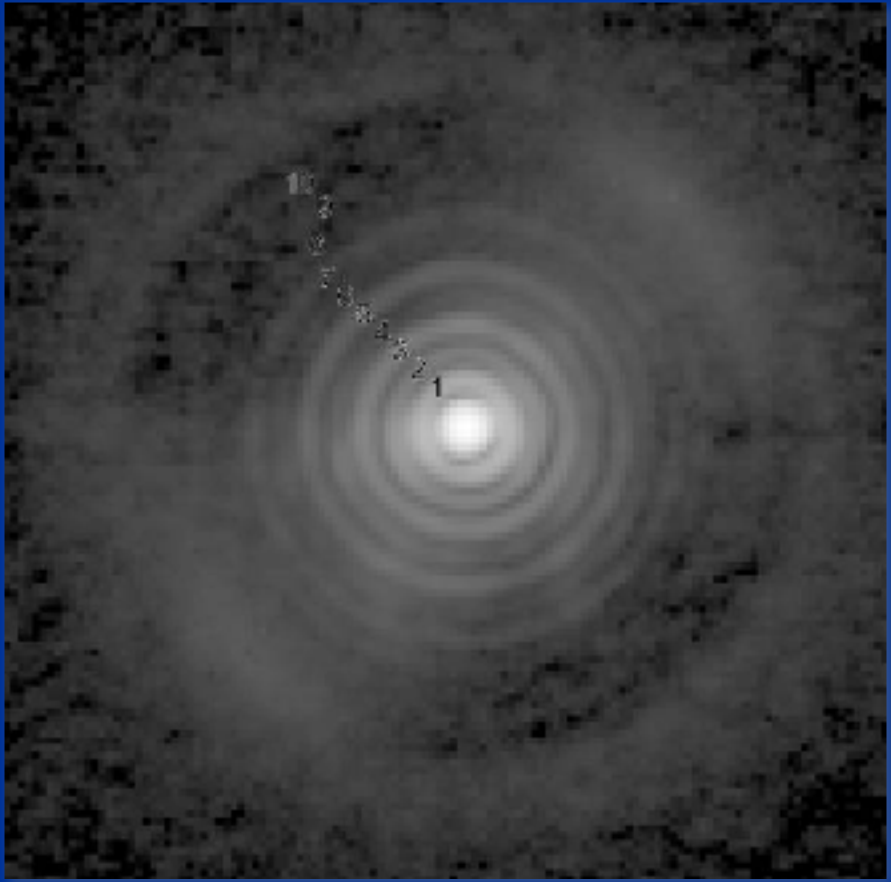
Adaptive secondary DMs have inherently high stroke: no need for separate tip-tilt mirror!

Adaptive secondary mirrors

- Advantages:
  - No additional mirror surfaces
    - » Lower emissivity. Ideal for thermal infrared.
    - » Higher reflectivity. More photons hit science camera.
  - Common to all imaging paths except prime focus
  - High stroke; can do its own tip-tilt

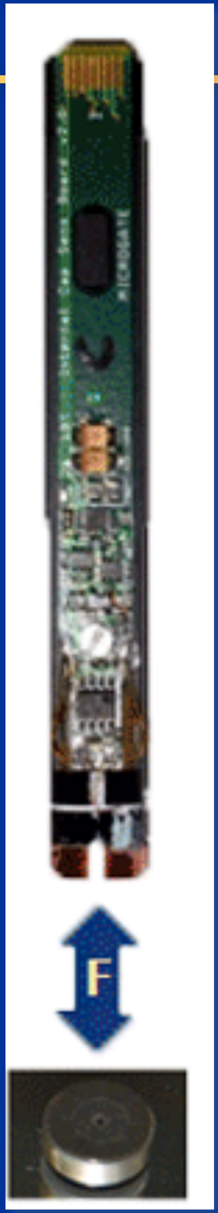
- Disadvantages:
  - Harder to build: heavier, larger actuators, convex.
  - Harder to handle (break more easily)
  - Must control mirror’s edges (no outer “ring” of actuators outside the pupil)
  - Current generation actuator are inefficient (each actuator requires ~0.3 W)

It Works! 10 Airy rings on the LBT!

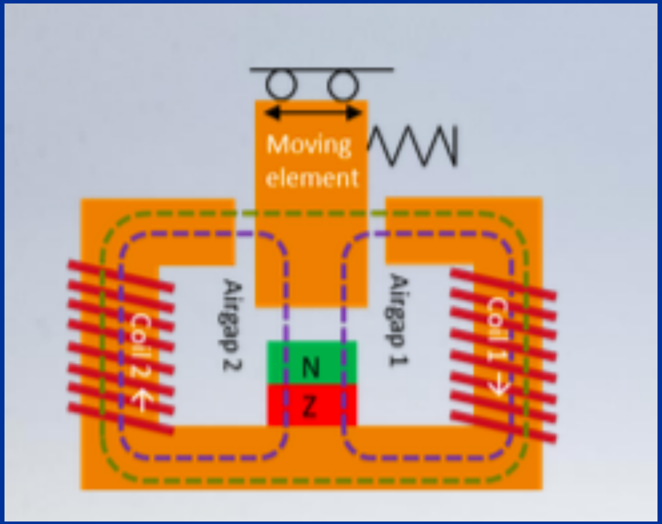


- Strehl ratio > 80%

Voice Coil actuators



- Current generation: fixed coil pushes and pulls on moving magnet
    - Most of magnetic flux is in air, reducing efficiency
- TNO has developed a “variable reluctance” design that greatly improves the efficiency (perhaps requiring ~0.03 W /actuator)



Concept Question

- Assume that its adaptive secondary mirror gives the 6.5 meter MMT telescope's AO system twice the throughput (optical efficiency) as conventional AO systems.
  - Imagine a different telescope (diameter D) with a conventional AO system.
  - For what value of D would this telescope+AO system have the same light-gathering power as the MMT?

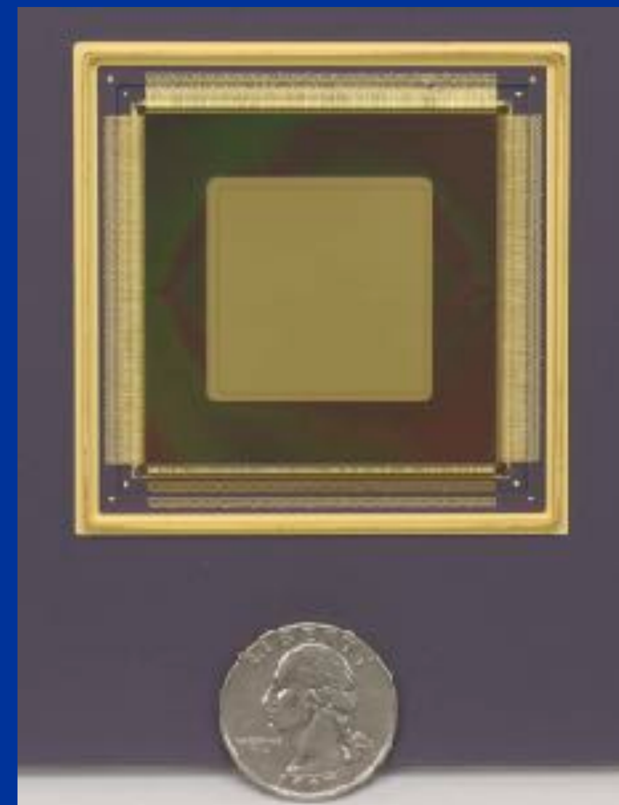
Cost scaling will be important for future giant telescopes

- Conventional DMs
  - About $1000 per degree of freedom
  - So $1M for 1000 actuators
  - Adaptive secondaries cost even more.
    - » VLT adaptive secondaries in range $12-14M each
- MEMS (infrastructure of integrated circuit world)
  - Less costly, especially in quantity
  - Currently ~ $100 per degree of freedom
  - So $100,000 for 1000 actuators
  - Potential to cost 10's of $ per degree of freedom

What are MEMs deformable mirrors?

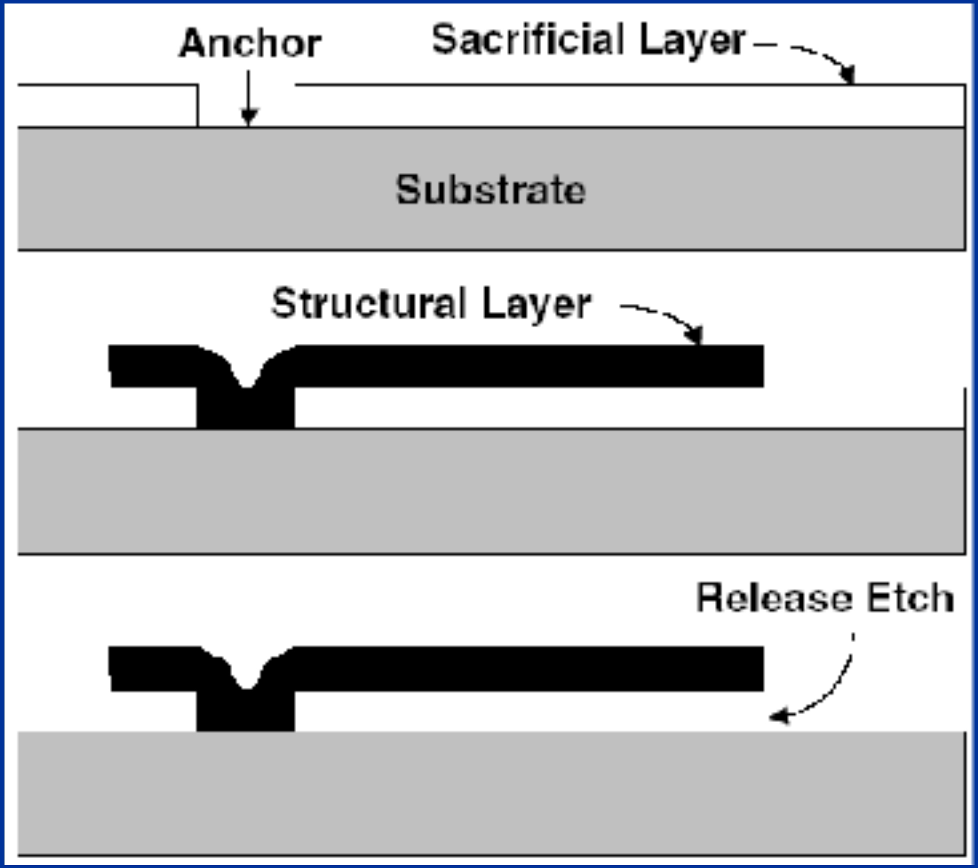
MEMS: Micro-electro-mechanical systems

- A promising new class of deformable mirrors, MEMs DMs, has recently emerged
- Devices fabricated using semiconductor batch processing technology and low power electrostatic actuation
- Potential to be less expensive ($10 - $100/actuator instead of $1000/actuator)



4096-actuator MEMS deformable mirror. Photo courtesy of Steven Cornelissen, Boston Micromachines

One MEMS fabrication process: surface micromachining

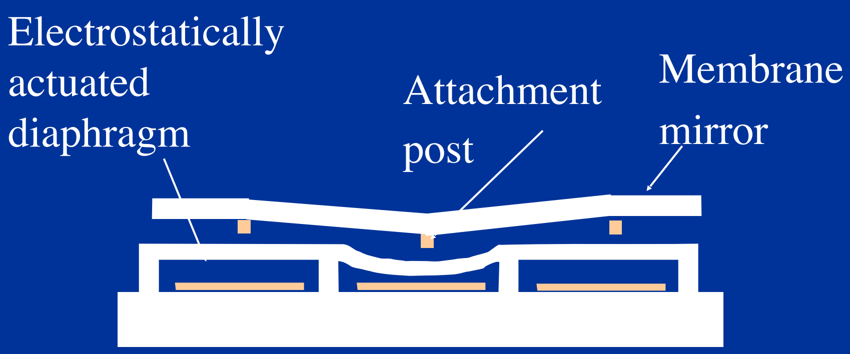


1

2

3

Boston University MEMS Concept



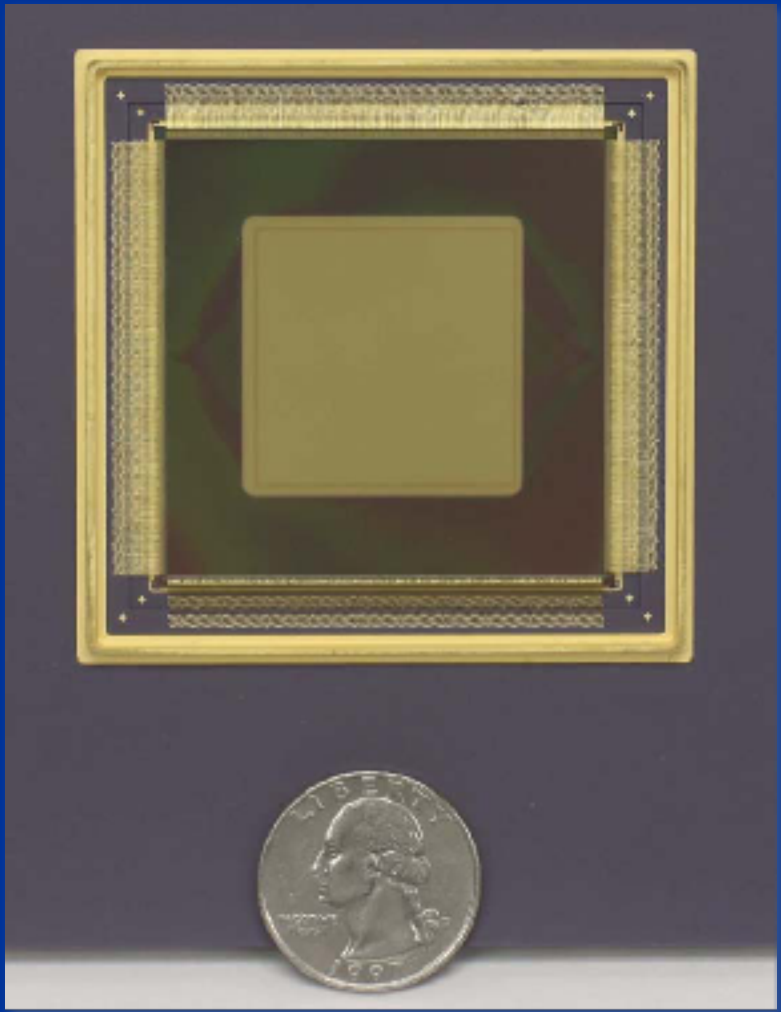
Continuous mirror

- Fabrication: Silicon micromachining (structural silicon and sacrificial oxide)
- Actuation: Electrostatic parallel plates

Boston University
Boston MicroMachines

Boston Micromachines: 4096 actuator MEMS DM

- Mirror for Gemini Planet Imager
- 4096 actuators
- 64 x 64 grid
- About 2 microns of stroke



MEMS testing: WFE < 1 nm rms in controlled range of spatial frequencies

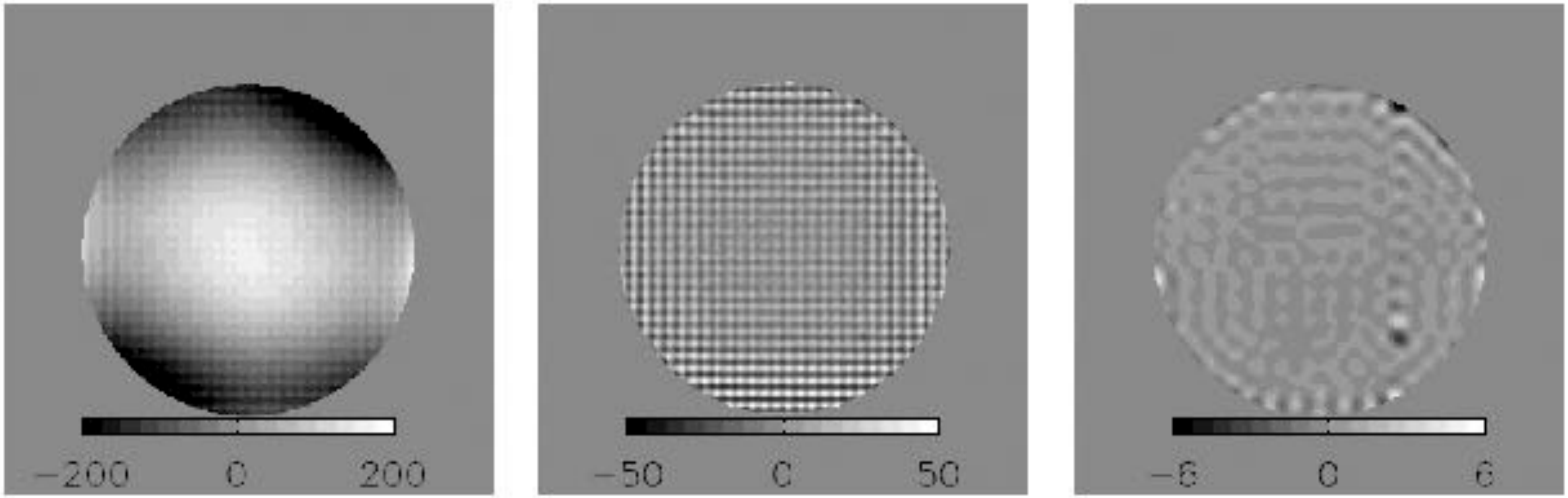
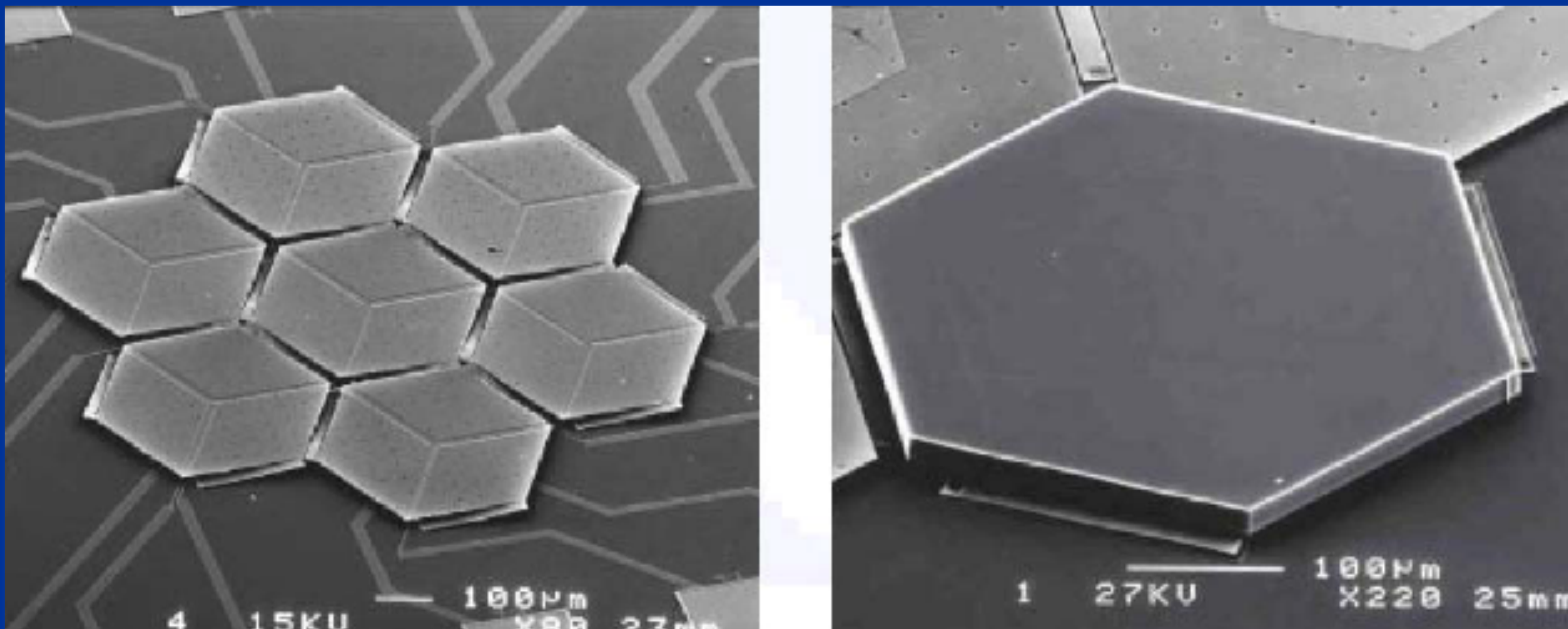


Figure 2. Wavefronts taken before and after a closed loop test with a 9.2 mm aperture. The initial wavefront has an RMS wavefront error of 148 nm (left), while the flattened wavefront has 12.8 nm total RMS wavefront error (center), which is mostly errors on the scale on an individual actuator. Inside the controlled range of spatial frequencies the RMS wavefront error is 0.54 nm. This is seen more clearly in the low-pass filtered image (right).

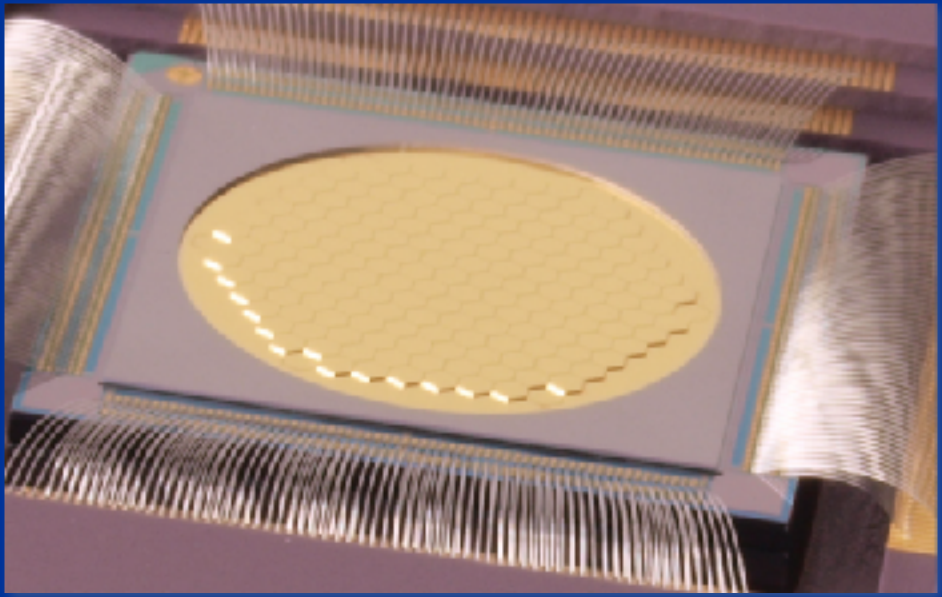
Credit: Morzinski, Severson, Gavel, Macintosh, Dillon (UCSC)

Another MEMS concept: IrisAO's segmented DM



- Each segment has 3 degrees of freedom
- Now available with 100's of segments
- Large stroke: > 7 microns

- IrisAO PT489 DM
- 163 segments, each with 3 actuators (piston+tip+tilt)
- Hexagonal segments, each made of single crystal silicon
- 8 microns of stroke (large!)



Issues for all MEMS DM devices

- "Snap-down"
  - If displacement is too large, top sticks to bottom and mirror is broken (can't recover)
- Robustness
  - Sensitive to humidity (seal using windows)
- Defect-free fabrication
  - Current 4000-actuator device still has quite a few defects

Concept Question

- How does the physical size (i.e. outer diameter) of a deformable mirror enter the design of an AO system?
  - Assume all other parameters are equal: same number of actuators, etc.

Fitting errors for various DM designs

$$\sigma_{fitting}^2 = \mu \, (d / r_0)^{5/3} \text{ rad}^2$$

DM Design	μ	Actuators / segment
Piston only, square segments	1.26	1
Piston+tilt, Square segments	0.18	3
Continuous DM	0.28	1

Consequences: different types of DMs need different actuator counts, for same conditions

- To equalize fitting error for different types of DM, number of actuators must be in ratio

$$\left(\frac{N_1}{N_2}\right) = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{a_{F_1}}{a_{F_2}}\right)^{6/5}$$

- So a piston-only segmented DM needs
  $(1.26 / 0.28)^{6/5} =$ 6.2 times more actuators than a continuous face-sheet DM!

- Segmented mirror with piston <u>and</u> tilt requires 1.8 times more actuators than continuous face-sheet mirror to achieve same fitting error: $N_1 = 3N_2 \,(0.18 / 0.28)^{6/5} = 1.8\, N_2$

Characterizing a Deformable Mirror

- A deformable mirror is a physical device that must be characterized to fit a wavefront accurately.
- Typical approach is to measure a deformable mirror with a phase shifting interferometer.
- Each actuator is "poked" and the resulting influence on the wavefront is measured.
    - These measurements are called the influence functions of the DM.
- These measurements are typically converted to a Zernike or similar basis set to be able to drive the mirror accurately.

Summary of main points

- Deformable mirror acts as a “high-pass filter”
  - Can’t correct shortest-wavelength perturbations
- Different types of mirror have larger/smaller fitting error
- Large DMs have been demonstrated (continuous face sheet, adaptive secondary) for ~ 1000 - 3000 actuators
- MEMs DMs hold promise of lower cost, more actuators
- Deformable secondary DMs look very promising
  - No additional relays needed (no off-axis parabolas), fewer optical surfaces
  - Higher throughput, lower emissivity
  - Early versions had problems; VLT has re-engineered now